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Applications of Arithmetic Mean and Geometric Mean in Statistical Analysis

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Arithmetic Mean

This article aims to highlight the situations in which various statistical measures can be effectively used, beginning with the mean. The definition of the arithmetic mean is the sum of the values of the same kind divided by the number of values observed. If the values plotted on graph paper form approximately a bell-shaped curve, then the best central measure to describe the data is the **arithmetic mean**. In general, almost all manufactured products follow a bell-shaped curve. In this case, the arithmetic mean is preferred as a measure of central value. General form of arithmetic mean is given as:

$$\overline{X} = \frac{\Sigma x}{n}$$

The arithmetic mean is used to compare the average performance of sales teams, provided there are no outliers. An outlier or extreme value is a numerical value that lies beyond the series of observation. For example, in the series of observations **{8, 18, 10, 12, 14, 16, 17, 37}**, 37 is an extreme value and is addressed as an outlier. It is important to remove the outlier from a series of observation to avoid skewness. Skewness refers to the condition where the data points tilt to any one side of observations. It can be understood better with the following instance - Average of the first seven values is **13.57**, whereas the average of all eight values (including the extreme value) is **16.5**. The average has increased by 21.59%, this difference of 21.59% from the original mean is due to the inclusion of extreme value. Outliers distort the arithmetic mean, and they should be eliminated if there is a sound reason.

Arithmetic mean and its applications are not only implemented in text books but is also useful in day-to-day life. For instance, Mr. Vivek owns a small workshop with 60 employees. This year, he earns 20% more profit than usual and expects this trend to continue for the next few years. Due to the advent of new industries in his locality, population has raised and so has the sales. To meet the new demand, he wants to motivate his employees to work more by increase their salaries by 10%. In this case, if the previous average salary was ₹10,000, and you are required to compute the new average salary after 10% increment in the employee salaries. To know the present average salary of employees, simply multiply the former average salary by 110, and divide by 100:

$$\textbf{New average salary} = 10{,}000 \times \frac{110}{100}$$

Therefore, the new average salary is ₹11,000.

In another instance, Mr. Suresh has five salesmen working under him, each assigned to different areas. Due to the varying population compositions and competitor strategies in each area, the average sales differ. As a true

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manager, it is difficult for him to select the best salesman. He decides to assign each salesman to an area with identical conditions. The report on the average sales of each salesman in that area gives the desired result. He can then plan the type of training to provide to other salesmen.

The application of the average can also be applied to the performance of students in different sections of the same stream under normal conditions. Share prices change over time, and companies quote average prices for reference. The travel time taken to reach the office from home varies daily, and we leave home based on the average time taken. The average life expectancy indicates the efforts made by the government towards healthcare. A higher value suggests excellent work by the government in the field of healthcare. It also helps compare the position of the country with respect to other countries in this field.

The position of the mean with respect to the median will indicate whether the distribution of values is normal, negatively skewed, or positively skewed. If the mean is less than the median, it is negatively skewed. If the mean is greater than the median, it is positively skewed.

Suppose we want to introduce a masala product with three ingredients: A, B, and C. The cost per gram of these ingredients is ₹5, ₹10, and ₹8, respectively. The product requires 2 grams of ingredient A, 5 grams of ingredient B, and 3 grams of ingredient C. The ingredient cost for 10 grams of masala will be ₹84 {A - (2x5) + B - (5x10) + C - (3x8) = 84}. Adding an operational cost of ₹16 for 10 grams and a profit margin of 20%, the total cost for 10 grams will be ₹120. In this case, the average cost per gram of ingredient would be ₹12, based on the weighted average.

The grades given to students by Universities are also based on weighted averages, where different subjects carry different weights. The Nifty Index is a combination of the weighted average of the top fifty companies based on their market capitalization, where the number of shares held by the company plays a role in the weight. Other well-known applications of the mean include ***average monthly family expenses, average distance covered per liter of petrol or diesel, and average time taken to finish a unit of product by a standard workman.***

A manufacturer has a machine that produces products ranging within 14 units to 18 units, with a mean value of 16 units. This implies **6 Sigma = 4**, so **Sigma = 2/3**. Suppose the consumer rejects the product if its value is below 15 or above 17.5. Under existing conditions, the rejection percentage on the lower side will be 6.68% and on the upper side 1.22%, totaling to 7.9%. With the help of standard deviation, the manufacturer can set the mean such that the percentage of product rejection is reduced. The rejection percentage can be reduced to 5% by shifting the mean to 16.3, which is always possible through machine adjustment.

There are situations where the average is not a suitable measure for decision-making. Suppose a manager has to promote one of two employees and has the following performance index for the last five years:

- Employee A: 17, 16, 21, 25, 26

- Employee B: 29, 27, 21, 17, 16

The average performance index for Employee A is 21, and for Employee B is 22. Employee B should not be selected based solely on the average. His performance index is gradually decreasing, whereas Employee A's performance index is increasing. Therefore, Employee A should be selected. The trend plays an important role in this decision.

A company recruits employees over the last ten years as follows: 30, 32, 28, 40, 39, 36, 35, 37, 34, 34. This indicates that the company recruits an average of 34.5 employees per year. However, it should be 34 or 35 and cannot be 34.5 in reality. This is similar to saying that the number of children in a household is 2.5. In these cases, reality plays a significant role.

Weighted Average

For example, suppose a Company makes 3 products X, Y and Z. They are expected to yield sales in the ratio 2:2:1. Actual contributions from X, Y and Z were found to be ₹1,00,000, ₹50,000 and ₹50,000 respectively and 5,000 units in aggregate were sold. The weighted average contribution margin is (1,00,000 x 2 + 50,000 x 2 + 50,000 x 1) = (3,50,000/5,000 = 70). This implies weighted average contribution margin is ₹70/unit.

Further if the company has fixed cost as ₹4,20,000, then the number of units to be sold to get brake even point is given by ₹4,20,000/70 = ₹6,000. Weighted average is used by stock investors to trace the cost basis of shares bought at different times. A wise investor will always compare EPS (earnings per share) metric of different company's share for investment. Therefore it is very important to know the correct way of calculating the EPS. Weighted average plays an important role in the calculation. The method is as follows.

A company as on January 1st, 2024, has 1,00,000 outstanding shares and issues an additional of 50,000 shares as on July 1st, 2024. The EPS is ₹1.25/share, calculated as follows. (1,00,000 x 12/12) + (50,000 x 6/12) = ₹1,25,000/1,00,000 = ₹1.25 for the year 2024. Further weighted average is used to calculate Portfolio Returns using allocation of assets as weights. Weighted average cost accounting is used to determine the cost of goods sold and value of ending inventory. Weighted average cost of capital is calculated by averaging the rate of all companies' sources of capital using proportion of each source as weight. This helps the investor to know the minimum rate of return.

Geometric Mean

Geometric Mean (GM) is another measure of central tendency of the distributions. It is the 'nth' root of products of 'n' variables. For example, GM of 8, 27, and 64 is 3rd root of 8 x 27 x 64 = 24 (2 x 3 x 4). Main advantage is that it gives low weight to extreme values and high weight to low values. It is the best measure of average for skewed distributions. Further, it is not affected by extreme values. In general it is used to calculate average of ratios, percentages and rates of increase or decrease. Construction of index number involve GM as it gives less weight to large numbers and more weight to smaller numbers. For an investment of ₹2,00,000 with interest rate of 6% in the first year, 8% interest second year and 10% in the third year, the average rate of interest is approximately 8.64% and not arithmetic mean of 6, 8 and 10 which is 8. Calculation of the average rate of interest does not require investment amount. It is given by **(1+0.06)(1+0.08)(1+0.10) = 0.0864.**

If rates of interest varies over years, then arithmetic mean will not be the actual rate of interest per year. This method of calculation is more useful if the investment amount is very high and for a longer period. We can compare different investment options with the help of Geometric average return without knowing the value of each, especially with respect to foreign currency investments.

The Compounded Annual Growth Rate (GAGR) is based on Geometric mean. GM helps to assess the return and growth. The movement of stock indices and financial securities can be predicted as they deal with percentages and growth rates.

The GM and Statistical Threshold Value (STV) is calculated from a number of generic E-Coli that causes infection, present in water samples. STV is a measure of variability for water distributions. GM is used to average the bacterial growth in microbiology. Bacteria concentration levels vary from 10 to 10,000 fold over a given period of time in lognormal distribution of pharmacokinetic data. GM is useful in analyzing environmental chemicals in blood or urine. Their distribution is skewed and hence GM is the best average. GM summarizes disease incidence rates or exposure levels in epidemiological studies.

In Health Statistics, annual growth rate of cancer incidence across different regions is calculated using GM Accurate relative magnitudes of mortality rates between two populations for overall health status factors such as life expectancy, disease prevailing, access to healthcare system etc., to construct an index. GM is more useful in studying the spread of any viral infection such as Covid-19 and to estimate the average growth rate.