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Robust fuzzy control of hydro-turbine regulating system with time-varying parameters and random disturbances

This paper studies the feasibility of robust fuzzy control of hydro turbine regulating systems (HTRSs). First, a mathematical model of the HTRS is presented. Then, to accommodate nonlinear vibrations, a novel fuzzy control method is designed for the HTRS. The method was built to process time-varying parameters and random disturbances to ensure robustness. The stability conditions of the HTRS are given as a set of linear matrix inequalities, and a detailed mathematical proof is presented that is easy to implement. Finally, the validity and superiority of the proposed method are shown by numerical simulations.

Keywords: Hydro turbine regulating system, nonlinear control, random disturbances, robust fuzzy control, time-varying parameters

1.0 Introduction

As more attention is paid to the sustainable generation of power, hydropower plays an increasingly important role in the world's energy strategy [1]. It is well known that the hydro turbine regulating system (HTRS) is a key element in the safe and stable operation of a hydropower station. However, it is a high-coupling, non-minimum phased, complicated nonlinear system that may exhibit nonlinear oscillations under certain operating conditions [2–4]. Therefore, the dynamic analysis and nonlinear control of HTRSs has attracted much attention [5–7].

On the nonlinear dynamics of HTRSs, many investigations have been made [8–10]. However, on the control of HTRSs, relevant research reports are quite few. This is because nonlinear control theory is still in the initial stage of development and less mature than linear control theory. Until now, some nonlinear control methods have been proposed for the vibration control of many nonlinear systems, even chaotic systems; for example, sliding mode control [11], adaptive control [12], pinning control [13], and predictive control [14].

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Due to the strong ability of fuzzy techniques to transform a nonlinear system into a linearization combination [15, 16], many fuzzy schemes have been proposed for controlling nonlinear systems [17–19]. Linear matrix inequality is an effective mathematical tool, and some studies have applied it to the fuzzy control of nonlinear systems [20, 21]. Considering that time-varying parameters and random disturbances are common during HTRS operation, fuzzy control is robust and may resist perturbations well.

For the above reasons, in this research we try to apply fuzzy control to a HTRS. First, the mathematical model of a HTRS is presented, and the time domains of nonlinear vibrations are shown. Second, a robust fuzzy control method for HTRSs is proposed. Third, the stability conditions are given as a set of linear matrix inequalities, which are easy to implement. The designed fuzzy control scheme can deal with time-varying parameters and random disturbances, showing the scheme's robustness. Finally, simulations are carried out to verify the effectiveness and validity of the presented method.

Following is the structure of this paper. In Section 2, a mathematical model of the HTRS is presented. The design of the robust fuzzy controller is given in Section 3. Simulation results are described in Section 4. The conclusions are drawn in Section 5.

2.0 HTRS Model

The mathematical model of a hydro turbine regulation system can be represented as [22]:

$$\begin{cases} \dfrac{d\delta}{dt} = \omega_0 \omega \\ \dfrac{d\omega}{dt} = \dfrac{1}{T_{ab}}(m_t - D\omega - \dfrac{E_q^{'} V_s}{x_{d\Sigma}^{'}}\sin\delta - \dfrac{V_s^2}{2}\dfrac{x_{d\Sigma}^{'} - x_{q\Sigma}}{x_{d\Sigma}^{'} x_{q\Sigma}}\sin 2\delta) \\ \dfrac{dm_t}{dt} = \dfrac{1}{e_{qh}T_w}(-m_t + e_y y + \dfrac{ee_y T_w}{T_y} y) \\ \dfrac{dy}{dt} = -\dfrac{1}{T_y} y \end{cases} \quad (1)$$

where δ, ω, m_t, and y are the rotor angle deviation of the generator, the relative deviation of the rotational speed of the generator, the hydro turbine output incremental torque deviation, and the incremental deviation of the guide vane opening respectively, and the parameters are: $\omega_0 = 314$, $T_{ab} = 9.0s$, D = 2.0, $E'_q = 1.35$, $x'_{d\Sigma} = 1.15$, $x_{q\Sigma} = 1.474$, $T_w = 0.8s$, $T_y = 0.1s$, $V_s = 1.0$, $e_{qh} = 0.5$, $e_y = 1.0$, e = 0.7.

For convenience, we used x_1, x_2, x_3, and x_4 to replace δ, ω, m_t, and y, thus:

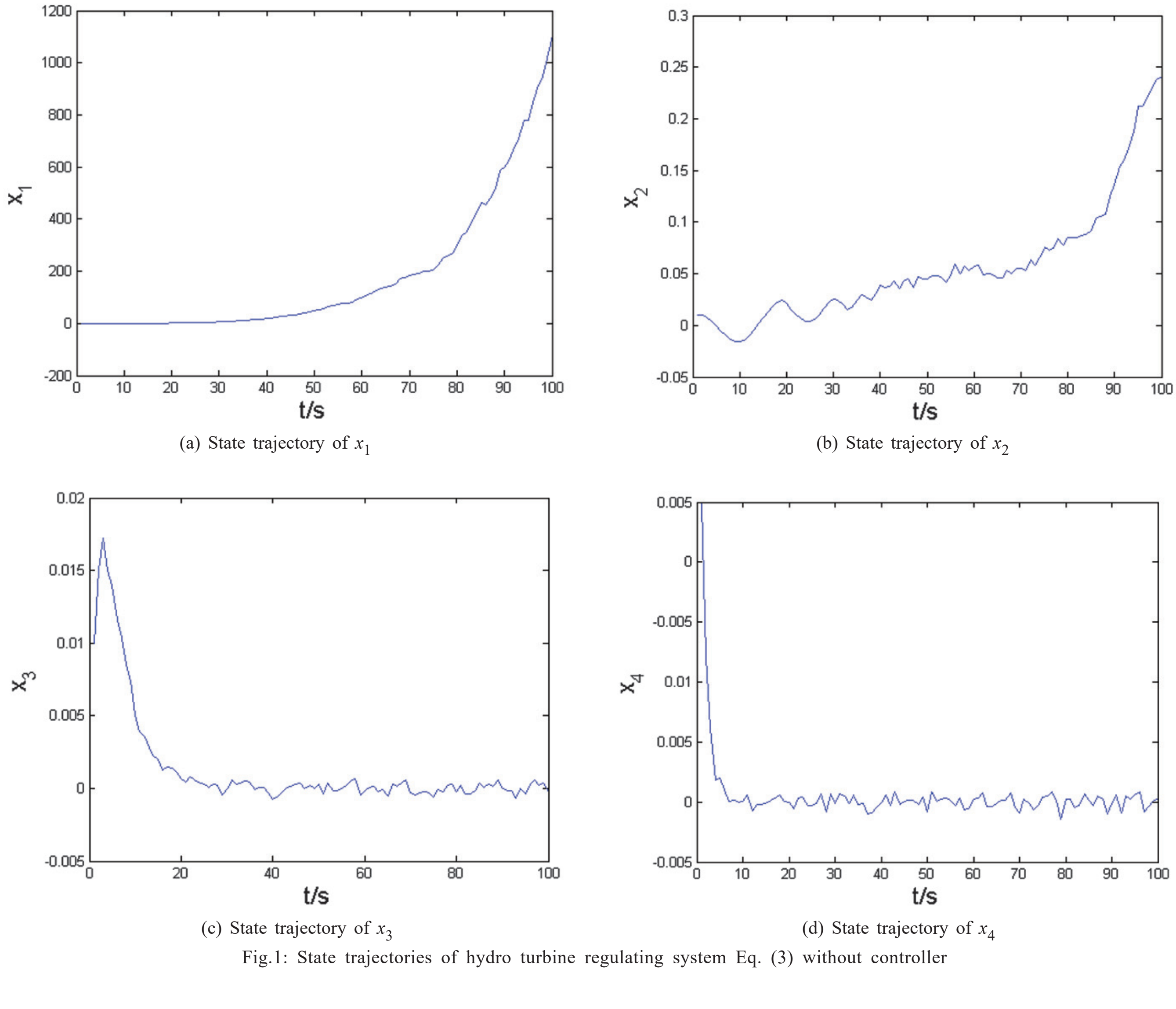
$$\begin{cases} \dfrac{dx_1}{dt} = \omega_0 x_2 \\ \dfrac{dx_2}{dt} = \dfrac{1}{T_{ab}}(x_3 - Dx_2 - \dfrac{E'_q V_s}{x'_{d\Sigma}}\sin x_1 - \dfrac{V_s^2}{2}\dfrac{x'_{d\Sigma} - x_{q\Sigma}}{x'_{d\Sigma} x_{q\Sigma}}\sin 2x_1) \\ \dfrac{dx_3}{dt} = \dfrac{1}{e_{qh}T_w}(-x_3 + e_y x_4 + \dfrac{ee_y T_w}{T_y}x_4) \\ \dfrac{dx_4}{dt} = -\dfrac{1}{T_y}x_4 \end{cases} \quad \text{... (2)}$$

When random disturbances are considered, then the HTRS Eq. (2) can be rewritten as:

$$\begin{cases} \dfrac{dx_1}{dt} = \omega_0 x_2 + rand(1)\times x_1 \\ \dfrac{dx_2}{dt} = \dfrac{1}{T_{ab}}(x_3 - Dx_2 - \dfrac{E'_q V_s}{x'_{d\Sigma}}\sin x_1 - \dfrac{V_s^2}{2}\dfrac{x'_{d\Sigma} - x_{q\Sigma}}{x'_{d\Sigma} x_{q\Sigma}}\sin 2x_1) + rand(1)\times x_2 \\ \dfrac{dx_3}{dt} = \dfrac{1}{e_{qh}T_w}(-x_3 + e_y x_4 + \dfrac{ee_y T_w}{T_y}x_4) + rand(1)\times x_3 \\ \dfrac{dx_4}{dt} = -\dfrac{1}{T_y}x_4 + rand(1)\times x_4 \end{cases} \quad \text{... (3)}$$

Figure 1 shows the state trajectories of the HTRS Eq. (3) with initial values

$[x_1\ x_2\ x_3\ x_4]^T = [0.01\ 0.01\ 0.01\ 0.01]^T$. Clearly, the system exhibit unstable operation and nonlinear vibrations, which need to be eliminated.



(a) State trajectory of x_1 (b) State trajectory of x_2

(c) State trajectory of x_3 (d) State trajectory of x_4

Fig.1: State trajectories of hydro turbine regulating system Eq. (3) without controller

3.0 Designing the robust fuzzy controller

3.1 INTERVAL MATRIX THEORY

When the uncertain parameters are considered, the HTRS Eq. (2) can be represented as:

$$\begin{cases} \frac{dx_1}{dt} = \tilde{a} x_2 \\ \frac{dx_2}{dt} = \tilde{b} x_3 - \tilde{c} x_2 - \frac{1}{T_{ab}} \times \frac{E_q^{'} V_s}{x_{d\Sigma}^{'}} \sin x_1 - \frac{1}{T_{ab}} \times \frac{V_s^2}{2} \frac{x_{d\Sigma}^{'} - x_{q\Sigma}}{x_{d\Sigma}^{'} x_{q\Sigma}} \sin 2x_1 \\ \frac{dx_3}{dt} = -\tilde{d} x_3 + \tilde{e} x_4 \\ \frac{dx_4}{dt} = \tilde{f} x_4 \end{cases}, \quad \text{... (4)}$$

where $\tilde{a} \in [a_1, a_2], \tilde{b} \in [b_1, b_2], \tilde{c} \in [c_1, c_2], \tilde{d} \in [d_1, d_2]$, $\tilde{e} \in [e_1, e_2], \tilde{f} \in [f_1, f_2]$.

To discuss the parameter uncertainties of the coefficient matrix of the HTRS Eq. (4), the following definition and lemma are introduced.

Definition 1 [23]: For the following equation to describe the linear system:

$$\dot{x}(t) = \tilde{A} x(t), \quad \text{... (5)}$$

where $x(t) \in R^n$ and the elements in the state matrix $\tilde{A}$ is not entirely sure, which belong to certain interval, so $\tilde{A}$ is called interval matrix, and Eq. (5) is called the interval system. Generally, the interval matrix can be described as:

$$\tilde{A} \in N[P, Q] = \left\{ \tilde{A} \in R^{n \times n} \mid p_{ij} \le a_{ij} \le q_{ij}, i, j = 1, \cdots, n \right\}$$

Among them, P and Q are the lower and upper bounds respectively of the elements in matrix $\tilde{A}$.

Lemma 1 [23]: The interval matrix $\tilde{A} \in N[P, Q]$ can be equivalently described as:

$$\tilde{A} = A_{0i} + E \Sigma F, \quad \text{... (6)}$$

where

$$A_{0i} = \frac{1}{2}(P_i + Q_i),$$

$$H = (h_{ij})_{n \times n} = H_i = \frac{1}{2}(Q_i - P_i),$$

$$\Sigma_i \in \Sigma^* = \left\{ \Sigma \in R^{n \times n} \mid \Sigma = \text{diag}(\varepsilon_{11}, \cdots \left| \varepsilon_{ij} \right| \le 1, i, j = 1, \cdots, n \right\},$$

$$E = (\sqrt{h_{11}} e_1, \ldots, \sqrt{h_{1n}} e_1, \ldots, \sqrt{h_{n1}} e_n, \ldots, \sqrt{h_{nn}} e_n),$$

$$F = (\sqrt{h_{11}} e_1, \ldots, \sqrt{h_{1n}} e_n, \ldots, \sqrt{h_{n1}} e_1, \ldots, \sqrt{h_{nn}} e_n)^T,$$

Here, e_i(i=1, ..., n) is the ith column of the $n \times n$ unit matrix.

Clearly, for i and $\Sigma_i \in \Sigma^*$, one can get:

1) $\Sigma_i \Sigma_i^T = \Sigma_i^T \Sigma_i \le I$;

2) $EE^T = diag\left\{ \sum_{j=1}^{n} h_{1j}, \cdots, \sum_{j=1}^{n} h_{nj} \right\}$;

3) $F^T F = diag\left\{ \sum_{i=1}^{n} h_{i1}, \cdots, \sum_{i=1}^{n} h_{in} \right\}$.

3.2 PARALLEL DISTRIBUTED COMPENSATION (PDC) CONTROLLER

Rule R_i : IF $z_1(t)$ is M_{i1} and ... and $z_n(t)$ is M_{ip}

$$\text{THEN } \frac{dx(t)}{dt} = (\tilde{A}_i + \Delta A_i) x(t) + (B_i + \Delta B_i) u(t)$$

$$r = (1, 2, \cdots, r), \quad \text{... (7)}$$

where M_{ij} = 1, 2, ..., n) is the fuzzy set and r is the fuzzy rule number, $x(t) \in R^n$ is the state vector, $A_i \in R^{n \times n}$ and $z(t) = [z_1(t)$, $z_2(t)$, ..., $z_p(t)]$ are the premise variables, $\alpha (0 < \alpha < 1)$ are the fractional orders, and $u(t)$ is the control input. The total output of the Takagi–Sugeno fuzzy model is presented as:

$$\frac{dx}{dt} = \sum_{i=1}^{r} h_i(z(t))(A_i + \Delta A_i) x(t) + \sum_{i=1}^{r} h_i(z(t))(B_i + \Delta B_i) u(t), \quad \text{... (8)}$$

where

$$h_i(z(t)) = \frac{\prod_{j=1}^{n} M_{ij}(z_j(t))}{\sum_{i=1}^{r} \prod_{j=1}^{n} M_{ij}(z_j(t))} \ge 0, \quad \sum_{i=1}^{r} h_i(z(t)) = 1 \quad \text{... (9)}$$

where M_{ij} ($z_j(t)$) is the grade of membership of $z_j(t)$ in M_{ij}.

The Takagi–Sugeno fuzzy controller $u(t)$ is designed based on the PDC and represented as follows:

Rule R^i : IF $z_1(t)$ is M_{i1} and ... and $z_n(t)$ is M_{ip}

$$\text{THEN } u(t) = K_i x(t) \quad (i = 1, 2, \cdots, r). \quad \text{... (10)}$$

The PDC controller output is designed in the following form:

$$u(t) = \sum_{i=1}^{r} h_i(z(t)) K_i x(t), \quad \text{... (11)}$$

where K_i (i = 1, 2, ..., r) represents the feedback gain.

By substituting (11) to (8), there follows:

$$\frac{dx}{dt} = \sum_{i=1}^{r} h_i(z(t))(A_i + \Delta A_i) x(t) + \sum_{i=1}^{r} h_i(z(t))(B_i + \Delta B_i) \sum_{i=1}^{r} h_i(z(t)) K_i x(t) \quad \text{... (12)}$$

According to the term $\sum_{i=1}^{r} h_i(z(t)) = 1$ in (9), (12) can be equally rewritten as

$$\frac{dx}{dt} = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t))(A_i + \Delta A_i + (B_i + \Delta B_i) K_j) x(t) \quad \text{...(13)}$$

3.3 Takagi–Sugeno fuzzy model of the HTRS

For the convenience of the coefficient matrix, the Maclaurin series expansion is introduced:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots \qquad \text{... (14)}$$

Assume $x_2 \in [-d, d]$, here $d = 2$. The Takagi–Sugeno fuzzy model is established, with the following two rules to describe the dynamic behavior of the system:

$$R^1: \text{IF } x_2 \text{ is } M_1(x_2(t)), \text{ THEN } \frac{dx(t)}{dt} = (\tilde{A}_1 + \Delta A_1)x(t) + (B_1 + \Delta B_1)u(t),$$

$$R^2: \text{IF } x_2 \text{ is } M_2(x_2(t)), \text{ THEN } \frac{dx(t)}{dt} = (\tilde{A}_2 + \Delta A_2)x(t) + (B_2 + \Delta B_2)u(t).$$

The membership functions are taken as follows:

$$M_1(x_2(t)) = \frac{1}{2}(1 + \frac{x_2(t)}{d}), \quad M_2(x_2(t)) = \frac{1}{2}(1 - \frac{x_2(t)}{d}).$$

In view of the above description, one can obtain:

$$\tilde{A}_1 = \begin{bmatrix} 0 & \tilde{a} & 0 & 0 \\ \frac{17231}{16951} & -\tilde{b} & \tilde{c} & 0 \\ 0 & 0 & -\tilde{d} & \tilde{e} \\ 0 & 0 & 0 & -\tilde{f} \end{bmatrix}, \tilde{A}_2 = \begin{bmatrix} 0 & \tilde{a} & 0 & 0 \\ \frac{1577}{16951} & -\tilde{b} & \tilde{c} & 0 \\ 0 & 0 & -\tilde{d} & \tilde{e} \\ 0 & 0 & 0 & -\tilde{f} \end{bmatrix},$$

$B_1 = B_2 = I_{4\times 4}$.

So the HTRS Takagi–Sugeno fuzzy model can be represented as

$$\dot{x}(t) = \sum_{i=1}^{2} h_i(z(t))(\tilde{A}_i + \Delta A_i + (B_i + \Delta B_i)u(t)) \qquad \text{... (15)}$$

3.4 Controller design

According to the PDC law, there is

$$u(t) = \sum_{j=1}^{2} h_j(z(t))K_j x(t)$$

Then, Eq. (15) could be obtained as:

$$\dot{x}(t) = \sum_{i=1}^{2}\sum_{j=1}^{2} h_i(x(t))h_j(x(t))(\tilde{A}_i + \Delta A_i + (B_i + \Delta B_i)K_j)x(t) \qquad \text{... (16)}$$

Based on the interval matrix theory in 3.1, (16) could be equivalently written as

$$\dot{x}(t) = \sum_{i=1}^{2}\sum_{j=1}^{2} h_i(x(t))h_j(x(t))(A_{0i} + E\Sigma F + \Delta A_i + (B_i + \Delta B_i)K_j)x(t) \qquad \text{... (17)}$$

Assumption 1: The parameter uncertainties considered here are norm bounded in the following form:

$$\Delta A_i = D_i F_i(t) E_{1i},$$
$$\Delta B_i = D_i F_i(t) E_{2i}.$$

where D_i, E_{1i}, E_{2i} are known real constant matrices of appropriate dimensions, F_i is a diagonal random matrix with Lebesgue-measurable elements and satisfies $F_i F_i^T \le I$, I is the identity matrix of appropriate dimension.

According to Assumption 1, (17) could be written as

$$\dot{x}(t)=\sum_{i=1}^{2}\sum_{j=1}^{2}h_i(x(t))h_j(x(t))(A_{0i}+E\Sigma F+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_j)x(t) \quad \text{... (18)}$$

Lemma 2 [24]: For any matrices ξ and θ, each with an appropriate dimension, and a random constant η>0, the following inequality holds:

$$2\xi^T\Sigma\theta\le\eta\xi^T\xi+\eta^{-1}\theta^T\theta, \quad \forall\Sigma\in\Sigma^* \quad \text{... (19)}$$

Theorem 1: If there exist a positive definite matrix P and a constant η, and the controller gain matrix K_i (i = 1, 2) is selected, which satisfies the following linear matrix inequalities, the HTRS Eq. (18) is globally asymptotically stable.

$$\begin{bmatrix}\Phi_{ii} & QF^T\\ FQ & \eta^{-1}I\end{bmatrix}<0 \quad \text{... (20)}$$

$$\begin{bmatrix}\Phi_{ij} & QF^{\ T}\\ FQ & -\eta^{-1}I\end{bmatrix}<0 \quad \text{... (21)}$$

$$Q>0 \quad \text{... (22)}$$

where $\Phi_{ii}=Q(A_i+D_iF_iE_{1i})^T+M_i^T(B_i+D_iF_iE_{2i})^T+(A_i+D_iF_iE_{1i})Q+(B_i+D_iF_iE_{2i})M_i+\eta EE^T$,

$$\Phi_{ij}=Q(A_i+D_iF_iE_{1i})^T+M_j^T(B_i+D_iF_iE_{2i})^T+Q(A_j+D_jF_jE_{1j})^T+M_i^T(B_j+D_jF_jE_{2j})^T+$$
$$(A_i+D_iF_iE_{1i})Q+(B_i+D_iF_iE_{2i})M_j+(A_j+D_jF_jE_{1j})Q+(B_j+D_jF_jE_{2j})M_i+\eta EE^T,$$

$Q=P^{-1}$, $M_i=K_iP^{-1}$, $M_j=K_jP^{-1}$, and I is a 4×4 identity matrix.

Proof: Assume the Lyapunov function $V(x) = x^T Px$, then one has:

$$\dot{V}(x)=\dot{x}^T Px+x^TP\dot{x}$$
$$=\sum_{i=1}^{4}\sum_{j=1}^{4}h_ih_j(A_{0i}+E\Sigma F+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_j)^T x^TPx$$
$$+x^TP\sum_{i=1}^{4}\sum_{j=1}^{4}h_ih_j(A_{0i}+E\Sigma F+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_j)$$
$$=\sum_{i=1}^{4}\sum_{j=1}^{4}h_ih_jx^T\left\{\left[(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_j)^TP+F^T\Sigma E^TP\right]\right.$$
$$\left.+\left[P(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_j)+PE\Sigma F\right]x\right\}$$
$$=\sum_{i=1}^{4}h_i^2x^T\begin{bmatrix}(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_i)^TP+F^T\Sigma E^TP+\\ P(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_i)+PE\Sigma F\end{bmatrix}x$$
$$+\sum_{i<j}^{4}h_ih_jx^T\begin{bmatrix}(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_j)^TP+F^T\Sigma E^TP+\\ P(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_j)+PE\Sigma F\end{bmatrix}x$$
$$+\sum_{i>j}^{4}h_ih_jx^T\begin{bmatrix}(A_{0j}+D_jF_jE_{1j}+(B_j+D_jF_jE_{2j})K_i)^TP+F^T\Sigma E^TP+\\ P(A_{0j}+D_jF_jE_{1j}+(B_j+D_jF_jE_{2j})K_i)+PE\Sigma F\end{bmatrix}x$$
$$=\sum_{i=1}^{4}h_i^2x^T\left[(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_i)^TP+P(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_i)\right]x$$

According to the term there is:

$$=\sum_{i=1}^{4} h_i^2 x^T \left[(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_i)^T P + P(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_i) \right] x$$

$$+\sum_{i=1}^{4} h_i^2 x^T (F^T \Sigma E^T P + PE\Sigma F)\, x \;\; +2\sum_{i<j}^{4} h_i h_j x^T (F^T \Sigma E^T P + PE\Sigma F)\, x$$

$$+\sum_{i<j}^{4} h_i h_j x^T \begin{bmatrix} (A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_j)^T P + \\ P(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_j) \end{bmatrix} x$$

$$+\sum_{i>j}^{4} h_i h_j x^T \begin{bmatrix} (A_{0j}+D_jF_jE_{1j}+(B_j+D_jF_jE_{2j})K_i)^T P + \\ P(A_{0j}+D_jF_jE_{1j}+(B_j+D_jF_jE_{2j})K_i) \end{bmatrix} x$$

According to the term $\sum_{i=1}^{4} h_i^2 + 2\sum_{i<j}^{4} h_i h_j = 1$, there is:

$$\sum_{i<j}^{4} h_i h_j x^T \left[(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_j)^T P + P(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_j) \right] x$$

$$+\sum_{i>j}^{4} h_i h_j x^T \left[(A_{0j}+D_jF_jE_{1j}+(B_j+D_jF_jE_{2j})K_i)^T P + P(A_{0j}+D_jF_jE_{1j}+(B_j+D_jF_jE_{2j})K_i) \right] x$$

$$=\sum_{i<j}^{4} h_i h_j x^T \left\{ \begin{matrix} \left[(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_j) + (A_{0j}+D_jF_jE_{1j}+(B_j+D_jF_jE_{2j})K_i) \right]^T P + \\ P\left[(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_j) + (A_{0j}+D_jF_jE_{1j}+(B_j+D_jF_jE_{2j})K_i) \right] \end{matrix} \right\} x$$

$$=2\sum_{i<j}^{4} h_i h_j x^T \left\{ \begin{matrix} \left[\dfrac{(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_j) + (A_{0j}+D_jF_jE_{1j}+(B_j+D_jF_jE_{2j})K_i)}{2} \right]^T P + \\ P\left[\dfrac{(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{2i})K_j) + (A_{0j}+D_jF_jE_{1j}+(B_j+D_jF_jE_{2j})K_i)}{2} \right] \end{matrix} \right\} x$$

Let

$$G_{ii} = A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{1i})K_i \,,$$

$$G_{ij} = \frac{(A_{0i}+D_iF_iE_{1i}+(B_i+D_iF_iE_{1i})K_j) + (A_{0j}+D_jF_jE_{1j}+(B_j+D_jF_jE_{2j})K_i)}{2}.$$

So one gets

$$\dot{V}(x) = \sum_{i=1}^{2} h_i^2 x^T (G_{ii}^T P + PG_{ii})x \;+2\sum_{i<j}^{2} h_i h_j x^T (G_{ij}^T P + PG_{ij})x \;+x^T F^T \Sigma E^T Px + x^T PE\Sigma Fx \quad ...(23)$$

From Lemma 2, selecting $\xi^T = x^T PE$, $\theta = Fx$, one can obtain

$$2x^T PE\Sigma Fx \le \eta\, x^T PEE^T Px + \eta^{-1} x^T F^T Fx \quad ...(24)$$

Taking the transpose of both sides in (24), one gets

$$2x^T F^T \Sigma E^T Px \le \eta\, x^T PEE^T Px + \eta^{-1} x^T F^T Fx \quad ...(25)$$

According to (24) and (25), there is

$$x^T F^T \Sigma E^T Px + x^T PE\Sigma Fx \le \;\; \eta\, x^T PEE^T Px + \eta^{-1} x^T F^T Fx \quad ...(26)$$

By substituting (26) into (23), one has

$$\dot{V}(x) \le \sum_{i=1}^{2} h_i^2 x^T (G_{ii}^T P + PG_{ii})x \;+2\sum_{i<j}^{2} h_i h_j x^T (G_{ij}^T P + PG_{ij})x \;+\eta x^T PEE^T Px + \eta^{-1} x^T F^T Fx \quad ...(27)$$

Note that $\sum_{i=1}^{4} h_i^2 + 2\sum_{i<j}^{4} h_i h_j = 1$, so (27) can be represented as

$$\dot{V}(x) \le \sum_{i=1}^{2} h_i^2 x^T (G_{ii}^T P + PG_{ii} + \eta PEE^T P + \eta^{-1} F^T F)x + 2\sum_{i<j}^{2} h_i h_j x^T (G_{ij}^T P + PG_{ij} + \eta PEE^T P + \eta^{-1} F^T F)x \quad \text{... (28)}$$

Accordingly, (28) can be assured as long as the following inequalities hold:

$$G_{ii}^T P + PG_{ii} + \eta PEE^T P + \eta^{-1} F^T F < 0 \quad (i, j = 1, 2) \quad \text{... (29)}$$

$$G_{ij}^T P + PG_{ij} + \eta PEE^T P + \eta^{-1} F^T F < 0 \quad (i < j \le 2) \quad \text{... (30)}$$

That is to say, the HTRS Eq. (18) is globally asymptotically stable if (29) and (30) hold. According to Schur's theorem, (29) and (30) could be transformed to the standard form of the LMIs which are described as (20), (21), and (22). This completes the proof.

4.0 Simulations

Considering the HTRS Eq. (3) with the parameter uncertainty: $\tilde{a} = 314 + 0.1\sin(t)$, $\tilde{b} = \frac{2}{9} + 0.1\cos(t)$, $\tilde{c} = \frac{1}{9} + 0.1\sin(t)$, $\tilde{d} = \frac{5}{2} + 0.1\cos(t)$, $\tilde{e} = \frac{33}{2} + 0.1\sin(t)$, and $\tilde{f} = 10 + 0.1\cos(t)$.

Therefore,

$$\tilde{a} \in [313.9, 314.1], \tilde{b} \in [\frac{11}{90}, \frac{29}{90}], \tilde{c} \in [\frac{1}{90}, \frac{19}{90}], \tilde{d} \in [2.4, 2.6], \tilde{e} \in [16.4, 16.6], \tilde{f} \in [9.9, 10.1]$$

To take control the HTRS with uncertain parameters, we take $d = 2$. A_{0i} $(i = 1, 2)$, E and F could be calculated by Lemma 1. The value of Σ could also refer to Lemma 1.

$B_1 = B_2 = I_{4\times4}$, $D_1 = D_2 = E_{11} = E_{12} = E_{21} = E_{22} = I_{4\times4}$,

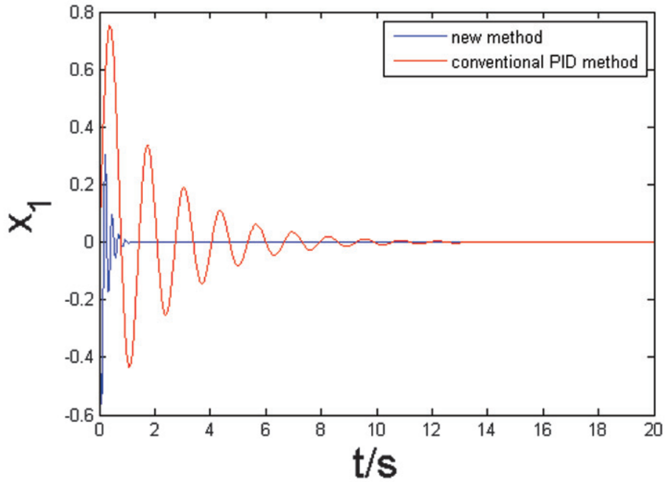
$F_1 = F_2$ = diag(diag(rand(4, 4))).

According to Theorem 1, and selecting η=1000, in the environment of Matlab7.0's LMI toolbox one gets

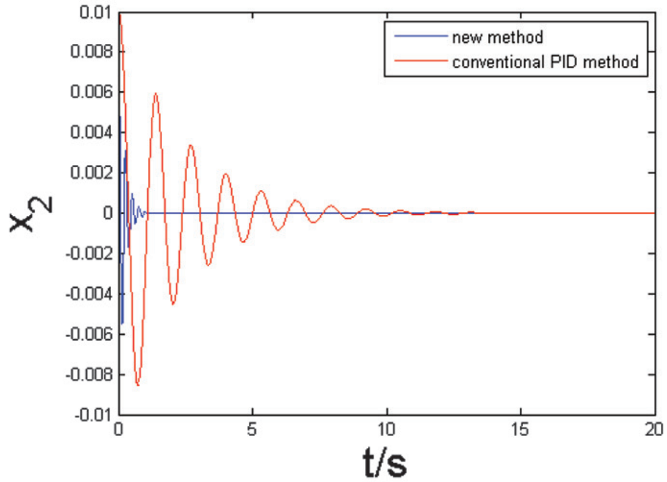
$$P = 10^{-4} \times \begin{bmatrix} 0.0000 & -0.0000 & 0.0000 & -0.0000 \\ -0.0000 & 0.1713 & 0.0010 & -0.0013 \\ 0.0000 & 0.0010 & 0.1819 & 0.0043 \\ -0.0000 & -0.0013 & 0.0043 & 0.1745 \end{bmatrix},$$

$$K_1 = 10^3 \times \begin{bmatrix} -0.0002 & -4.0410 & -0.0236 & 0.0297 \\ -0.0004 & -0.0094 & -0.0001 & 0.0000 \\ 0.0000 & 0.0000 & -0.0046 & -0.0059 \\ -0.0000 & -0.0001 & -0.0036 & -0.0011 \end{bmatrix},$$

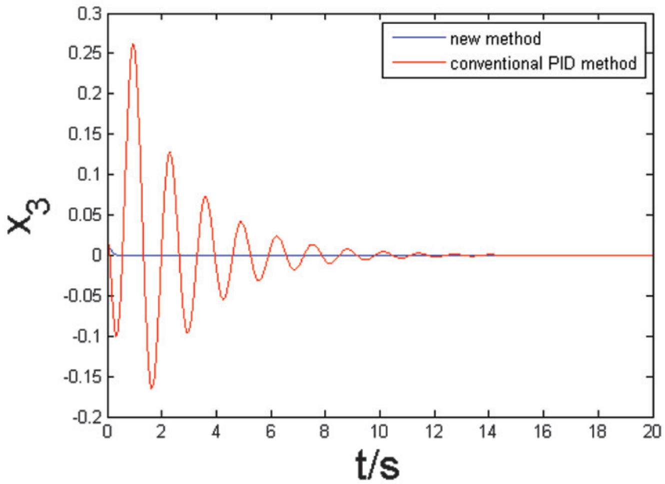
$$K_2 = \begin{bmatrix} -0.2875 & -503.5276 & -2.5156 & 2.7235 \\ -0.0437 & -9.2668 & -0.0588 & 0.0062 \\ 0.0001 & 0.0174 & -4.5846 & -5.8536 \\ -0.0001 & -0.0771 & -3.5710 & -1.0584 \end{bmatrix}.$$

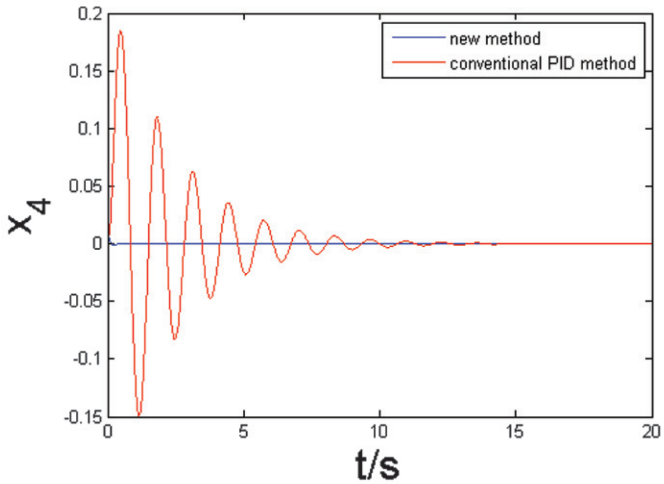


(a) State trajectory of x_1



(b) State trajectory of x_2





(c) State trajectory of x_3 (d) State trajectory of x_4

Fig.2: State trajectories of the controlled hydro turbine regulating system Eq.(3)

Fig.2 show the simulation results of the proposed method designed in this paper along with the traditional proportional–integral–derivative (PID) control method. Clearly, when the controller is applied to the HTRS, the system state variables are stabilized quickly, showing the effectiveness of the proposed method. By comparing it with the traditional PID control method, one can see that both the overshoot and adjustment time are small and short, showing the robustness and superiority of the designed scheme.

5.0 Conclusion

The robust fuzzy control of HTRSs was studied in this paper. First, a mathematical model of the nonlinear HTRS was presented. Second, a novel fuzzy control method was proposed for vibration control of the HTRS. Then the stability conditions of the HTRS were given as a set of linear matrix inequalities, and a detailed mathematical proof was presented. We found that the method could handle the case of time-varying parameters and random disturbances, which shows the good robustness. Finally, validity and superiority were verified by simulation results. In the future, the author will try to extend the scheme to fractional-order HTRS.

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