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Buckling analysis of thick plates using a single variable simple plate theory

Buckling analysis of thick plates has been carried out herein by using a single variable simple plate theory. Theory used herein is a third order shear deformation plate theory which uses a single displacement function for the complete formulation of plates. Plate formulation is governed by only one governing differential equation. Governing equation of the theory has close resemblance to that of Classical Plate Theory. Thus, plate problems can be solved in the similar lines as in case of classical plate theory. Plate theory used herein does not require a shear correction coefficient. To check the efficacy of the theory buckling analysis of simply supported thick rectangular plates is carried out. Critical buckling loads for simply supported plates are evaluated and the results obtained are compared to other shear deformation plate theories. Buckling load results are found to be in good agreement with other plate theory results.

Keywords: Buckling analysis, thick plates, higher order, shear deformation, simply supported plate.

1.0 Introduction

Buckling analysis of thin and thick plates is one of the important areas of investigation in the field of plate theory research. Buckling analysis of thin and thick plates has been discussed in literature in a comprehensive manner by using different class of plate theories. The important classical and shear deformation theories used commonly for the analysis of thin and thick plates are: Classical plate theory (involves one variable) (CPT) [1, 2], Mindlin's first order plate theory (involves three variables) [3], Reddy's higher order plate theory (involves three variables) [4, 5] and Refined plate theory (involves two variables) (RPT) [6].

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Classical plate theory (CPT) [1, 2] is the basic and oldest theory in the literature of plate theories. CPT equations do not include the shear deflection component in the formulation. This drawback restricts the use of CPT only for the investigation of thin plates. The use of CPT for the buckling analysis of thick plates will result in the overestimated buckling loads. Also, the CPT yields the overestimated values for frequencies and underestimated deflection values in case of thick plates. The inaccuracies in the predicted results would increase as the plate thickness increases. This drawback of CPT demands for the use of refined or higher order plate theories for the investigation of thick plates. A detailed study on thin plate formulation based upon CPT could be found in the textbooks by Timoshenko and Woinowsky-Krieger [1] and Timoshenko and Gere [2].

In the class of thick or shear deformation theories, the plate theory proposed by Mindlin is one of the oldest theories developed. Mindlin's theory is a displacement based first order plate theory [3]. The study of thick plates using Mindlin's theory linked with three displacement variables and the plate formulation requires three governing differential equations. In comparison with CPT results, Mindlin's theory can give considerably accurate results in case of thick or shear deformable plates [7]. The formulation of the theory involves a shear coefficient or shear correction factor. Shear coefficient is necessary to add correction to the values of transverse shear stresses evaluated by Mindlin's theory. Because, Mindlin's theory yields the constant transverse shear stress across the plate thickness instead of actual parabolic shear stress distribution. This is a common drawback involved in case of first order beam and plate theory formulation. Many research papers are available in the beam and plate theory literature providing discussion on the use of shear coefficients. Important papers available based on Mindlin's theory are: Wang and Alwis [8], Wang et. al [9], Wang et. al [10] and Lee et. al [11].

In the class of higher order plate theories, Reddy's theory is one of the most popular plate theories. Reddy's theory is a displacement based third order plate theory [4, 5]. Reddy's theory is governed by five coupled differential equations and involves five unknown displacement variables. Being a higher order theory, this theory does not involve a shear coefficient or shear correction factor in the plate formulation. The transverse shear stress distribution is parabolic across the plate thickness. Hence, the condition of shear free stress condition is automatically satisfied. Some of the important papers available based on Reddy's plate theory are: Reddy and Phan [12], Reddy and Wang [13], Shufrin and Eisenberger [14] and Hashemi et. al [15].

Refined plate theory (RPT) [6] is a displacement based two variable higher order plate theory. Due to the involvement of only two variables, the plate analysis using RPT is considerably simplified compared to other higher order theories. The formulation of the theory splits the lateral deflection into two components; bending component and shear component. RPT formulation leads to two coupled governing differential equations for vibration study. These equations are decoupled in case of static problems. The theory has strong similarity to CPT expressions. The moment and shear force expressions of RPT have strong resemblance to the CPT expressions. Important papers available based on RPT are reported in the publications by Thai and Choi [16], Thai and Kim [17, 18]. Other important works based on RPT are also available in the publications by Shimpi et al [19, 20].

Objective of this paper is to study the buckling analysis of thick shear deformable plates by using a third order Single Variable Simple Plate Theory (SVSPT) published in a paper by Shimpi et al. [21]. SVSPT used herein for the investigation of thick plates is developed based upon the formulation of RPT [6, 19, 20]. SVSPT incorporates a single displacement variable for the complete formulation of plates. Lateral deflection of the plate is the unknown displacement variable involved in the plate formulation. Displacement field and the expressions for strains, stresses are all can be expressed in terms of a single variable. Governing equation is a fourth order differential equation incorporating a single unknown function. Also, governing equation has close resemblance to that of CPT. Hence, the plate analysis using the theory used herein will be almost in the similar lines of that of CPT. Efforts involved in solving the plate problems using SVSPT is slightly more compared to that of CPT. Venkatesha B K et al. [25, 26] studied the numerical analysis of damage tolerance design. Fatigue crack growth rate and stress intensity factor range was estimated with Paris law of damage crack growth.

In this paper, the usefulness of SVSPT is showcased by carrying out the buckling study of thick plates. Plates with simply supported edge conditions are considered for the discussion. Buckling loads calculated by SVSPT are compared with the buckling loads predicted by CPT and other thick plate shear deformation theories for the validation purpose. Buckling load results are presented in a tabular form for the easy comparison and observation.

2.0 Plate formulation: single variable simple plate theory (SVSPT)

Displacement, bending moment and shear force expressions of SVSPT will be presented now in this section. Also, the boundary conditions and governing equation pertaining to SVSPT will also be discussed. The more details about single variable simple plate theory is available in a publication on SVSPT by Shimpi et al. [21].

2.1 Displacement field of SVSPT

Displacement field of SVSPT is discussed herein. The axial or in-plane displacements (u and v) and lateral deflection (w) of SVSPT are as follows [21]:

$$u = -z\frac{\partial w_b}{\partial x} + \frac{2(1+\mu)}{E}\left[\frac{3}{10}\left(\frac{z}{h}\right) - 2\left(\frac{z}{h}\right)^3\right] \left[-D\frac{\partial}{\partial x}\left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_b}{\partial y^2}\right)\right] \quad \text{... (1)}$$

$$v = -z\frac{\partial w_b}{\partial y} + \frac{2(1+\mu)}{E}\left[\frac{3}{10}\left(\frac{z}{h}\right) - 2\left(\frac{z}{h}\right)^3\right] \left[-D\frac{\partial}{\partial y}\left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_b}{\partial y^2}\right)\right] \quad \text{... (2)}$$

$$w = w_b + \frac{12(1+\mu)}{5Eh}\left[-D\left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_b}{\partial y^2}\right)\right] \quad \text{... (3)}$$

Equations (1), (2) and (3) are the displacement expressions pertaining to SVSPT. The displacements u, v and w contain only w_b as an unknown displacement variable. Therefore, the expressions for strains and stresses also contain single unknown displacement variable. For the strain and stress expressions of SVSPT one can refer to the publication by Shimpi et al. [21]. Further, the plate formulation using SVSPT is in the similar lines of CPT wherein also single variable is involved in the formulation [1].

2.2 Expressions for moments and shear forces of SVSPT

The bending moments (M_x, M_y, M_{xy}) and shear forces (Q_x, Q_y) given by SVSPT are as follows:

$$M_x = -D\left(\frac{\partial^2 w_b}{\partial x^2} + \mu\frac{\partial^2 w_b}{\partial y^2}\right) \quad \text{... (4)}$$

$$M_y = -D\left(\frac{\partial^2 w_b}{\partial y^2} + \mu\frac{\partial^2 w_b}{\partial x^2}\right) \quad \text{... (5)}$$

$$M_{xy} = -D\,(1-\mu)\,\frac{\partial^2 w_b}{\partial x\,\partial y} \quad \text{... (6)}$$

$$Q_x = -D\frac{\partial}{\partial x}\left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_b}{\partial y^2}\right) \quad \text{... (7)}$$

$$Q_y = -D\frac{\partial}{\partial y}\left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_b}{\partial y^2}\right) \quad \text{... (8)}$$

The bending moments (M_x, M_y, M_{xy}) and shear forces (Q_x, Q_y) are given by Eqs. (4) - (8). The expressions contain only w_b as an unknown displacement variable. The above expressions for moments and shear forces also have similar appearance as those of CPT expressions [1, 2].

2.3 Derivation for Governing Differential Equation

Geometry of the plate under consideration is as shown in Fig.1. The plate under consideration is subjected to the combined action of in-plane loads N_x (acting along x-direction), N_y (acting along y-direction), shearing forces N_{xy} (acting in xy plane) and a lateral distributed load of intensity $q(x, y)$ (acting along z-direction).

For the above set of loading condition, the plate equilibrium equations can be written as follows [21]:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \qquad \text{... (9)}$$

$$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0 \qquad \text{... (10)}$$

$$\frac{\partial^2 M_x}{\partial x^2} - 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -\left(q(x,y) + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2}\right) \qquad \text{... (11)}$$

Substituting for M_x, M_y and M_{xy} from Eqs.(4)-(6), respectively, one obtains

$$D\left[\frac{\partial^4 w_b}{\partial x^4} + 2\frac{\partial^4 w_b}{\partial x^2 \partial y^2} + \frac{\partial^4 w_b}{\partial y^4}\right] = q(x,y) + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \qquad \text{... (12)}$$

Equation (12) is considered as the governing differential equation for determining the deflection surface of a plate by considering the effects of in-plane forces N_x, N_y and N_{xy}. The above differential equation is linked with only one unknown displacement variable (w_b). In appearance, the governing equation (12) is closely similar to that of CPT governing equation [1, 2].

3.0 Simply supported rectangular plate: derivation of characteristic equation for plate buckling using SVSPT

Consider a rectangular plate as shown in Fig.1. Plate under consideration is subjected to simply supported boundary conditions at edges x=0, a and y=0, b. Plate is subjected to in-plane forces $N_x = N_y = -N_0$ and shearing force N_{xy} is taken as zero. Therefore, the governing equation (12) can be rewritten as follows:

$$D\left[\frac{\partial^4 w_b}{\partial x^4} + 2\frac{\partial^4 w_b}{\partial x^2 \partial y^2} + \frac{\partial^4 w_b}{\partial y^4}\right] = q(x,y) - 2N_0 \frac{\partial^2 w}{\partial x^2} \qquad \text{... (13)}$$

Plate is also subjected to a lateral distributed load $q(x,y)$. Lateral load $q(x,y)$ applied on the plate can be represented using double Fourier series as follows:

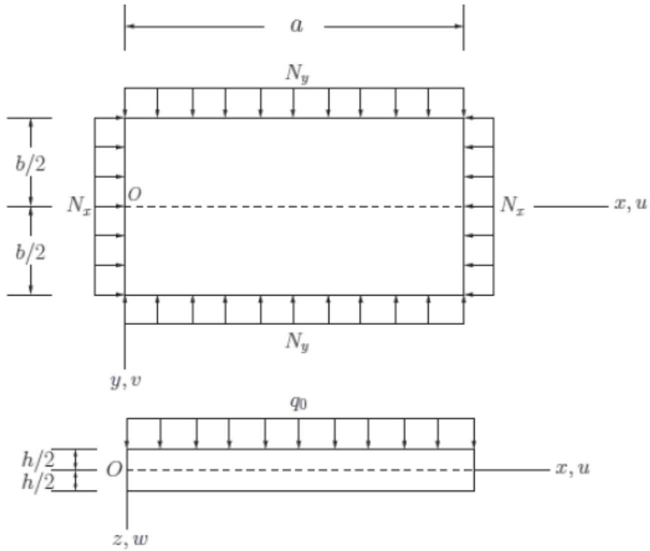

Fig.1: Rectangular plate under the combined action of in-plane and lateral loads

$$q(x,y) = \sum_{m=1,2,3..}^{\infty} \sum_{n=1,2,3..}^{\infty} q_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \qquad \text{... (14)}$$

where q_{mn} is a Fourier constant which depends upon type of loading. For lateral distributed load, Fourier constants are given by

$$q_0 = \frac{16 q_0}{\pi^2 mn} \quad \text{For } m=1,3,5, \ldots \text{ and } n=1,3,5, \ldots$$

q_0=0 For m=2,4,6, ... and n=2,4,6, ...

Navier's solution for w_b which can satisfy the simply supported boundary conditions at edges x=0, a and y=0,b can be written as follows:

$$w_b = \sum_{m=1,2,3,..}^{\infty} \sum_{n=1,2,3,..}^{\infty} C_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \qquad \text{... (15)}$$

where C_{mn} is the constant associated with displacement.

Substituting Eqs. (14) and (15) in governing equation (13), one can write the characteristics equation for the plate buckling as follows:

$$\frac{N_x a^2}{D}\left[m^2\pi^2 + \frac{1}{5(1-\mu)}\left(\frac{m\pi h}{a}\right)^2 \left(m^2\pi^2 + \left(\frac{n\pi a}{b}\right)^2\right)\right]$$
$$+ \frac{N_y a^2}{D}\left[\left(\frac{n\pi a}{b}\right)^2 + \frac{1}{5(1-\mu)}\left(\frac{h}{a}\right)^2 \left[m^4\pi^4 + 2\left(\frac{n\pi a}{b}\right)^4 + 3\left(\frac{n\pi a}{b}\right)^2 m^2\pi^2\right]\right]$$
$$+ \frac{N_x a^2}{D}\frac{N_y a^2}{D}\frac{1}{5(1-\mu)}\left(\frac{m\pi h}{a}\right)^2\left[1 + \frac{1}{5(1-\mu)}\alpha_{mn}\right]$$
$$+ \left(\frac{N_y a^2}{D}\right)^2 \frac{1}{5(1-\mu)}\left(\frac{n\pi h}{b}\right)^2\left[1 + \frac{1}{5(1-\mu)}\alpha_{mn}\right] + \left[m^2\pi^2 + \left(\frac{n\pi a}{b}\right)^2\right]^2 = 0 \qquad \text{... (16)}$$

where, $\alpha_{mn} = \left(\frac{m\pi h}{a}\right)^2 + \left(\frac{n\pi h}{b}\right)^2$

Rearranging the terms, Eq. (16) can also be written as follows:

$$\frac{N_x a^2}{\pi^2 D}\left[m^2+\frac{1}{5(1-\mu)}\left(\frac{m\pi h}{a}\right)^2\left(m^2+\left(\frac{na}{b}\right)^2\right)\right]+\frac{N_y a^2}{\pi^2 D}\left[\left(\frac{na}{b}\right)^2+\frac{\pi^2}{5(1-\mu)}\left(\frac{h}{a}\right)^2\left[m^4+2\left(\frac{na}{b}\right)^4+3\left(\frac{na}{b}\right)^2 m^2\right]\right]$$
$$+\frac{N_x a^2}{\pi^2 D}\frac{N_y a^2}{\pi^2 D}\frac{1}{5(1-\mu)}\left(\frac{m\pi h}{a}\right)^2\left[1+\frac{1}{5(1-\mu)}\alpha_{mn}\right] \quad \text{... (17)}$$
$$+\left(\frac{N_y a^2}{\pi^2 D}\right)^2\frac{1}{5(1-\mu)}\left(\frac{n\pi h}{b}\right)^2\left[1+\frac{1}{5(1-\mu)}\alpha_{mn}\right]+\left[m^2+\left(\frac{na}{b}\right)^2\right]^2=0$$

By substituting $N_x = N_y = -N_0$ in Eq. (17), we have

$$\left(\frac{N_0 a^2}{\pi^2 D}\right)^2\frac{1}{5(1-\mu)}\left(\left(\frac{m\pi h}{a}\right)^2+\left(\frac{n\pi h}{b}\right)^2\right)\times\left[1+\frac{1}{5(1-\mu)}\left(\left(\frac{m\pi h}{a}\right)^2+\left(\frac{n\pi h}{b}\right)^2\right)\right]$$
$$-\left(\frac{N_0 a^2}{\pi^2 D}\right)\left[\frac{2\pi^2}{5(1-\mu)}\left(\frac{h}{a}\right)^2(m^2+n^2)^2+(m^2+n^2)\right]+(m^2+n^2)^2=0 \quad \text{... (18)}$$

TABLE 1: RESULTS FOR BUCKLING COEFFICIENT (?) OF A SIMPLY SUPPORTED SQUARE PLATE UNDER BUCKLING LOADS

Theory	Results for buckling coefficient (?)			
	Uniaxial compression ($N_x = -N_0$ and N_y=0)			
	h/a=0.001	h/a=0.05	h/a=0.1	h/a=0.2
Exact [22]	4.000	3.9110	3.7410	3.1500
CPT [2]	4.000	4.0000	4.0000	4.0000
Mindlin [23]	4.000	3.9444	3.7864	3.2637
Reddy [24]	4.000	3.9443	3.7865	3.2653
RPT [16]	4.000	3.9443	3.7865	3.2653
SVSPT	4.000	3.9444	3.7864	3.2637
	Biaxial compression ($N_x = -N_0$ and $N_y = -N_0$)			
	h/a=0.001	h/a=0.05	h/a=0.1	h/a=0.2
CPT [2]	2.0000	2.0000	2.0000	2.0000
Mindlin [23]	2.0000	1.9722	1.8932	1.6319
Reddy [24]	2.0000	1.9722	1.8933	1.6327
RPT [16]	2.0000	1.9722	1.8933	1.6327
SVSPT	2.0000	1.9722	1.8932	1.6319

In case of square plates, Eq. (18) can be written as follows:

$$\frac{N_0 a^2}{\pi^2 D}=-\frac{\left[m^2+\left(\frac{na}{b}\right)^2\right]^2}{m^2\left[1+\frac{1}{5(1-\mu)}\left(\frac{h}{a}\right)^2\left(m^2\pi^2+\left(\frac{n\pi a}{b}\right)^2\right)\right]} \quad \text{... (19)}$$

It is evident that the value of critical buckling load is obtained by taking $n = 1$. Hence,

$$\lambda=\frac{N_{cr}\,a^2}{\pi^2 D}=\frac{\left[m^2+\left(\frac{a}{b}\right)^2\right]^2}{\left[m^2+\frac{1}{5(1-\mu)}\left(\frac{m\pi h}{a}\right)^2\left(m^2+\left(\frac{a}{b}\right)^2\right)\right]} \quad \text{... (20)}$$

In case of square plates, Eq. (20) can be written as follows:

$$\lambda=\frac{[1+m^2]^2}{\left[m^2+\frac{1}{5(1-\mu)}\left(\frac{m\pi h}{a}\right)^2(1+m^2)\right]} \quad \text{... (21)}$$

4.0 Results for buckling loads and discussion on the results

Table 1 presents the results for buckling coefficients (γ) of a square plate with simply supported edge conditions and subjected to compressive in-plane loads N_x and N_y. The buckling coefficient (γ) used in Table 1 is defined as follows:

$$\lambda=\frac{N_{cr}a^2}{\pi^2 D}$$

For the comparison of buckling loads calculated by SVSPT, Table 1 also presents the buckling loads predicted by using the CPT and other first order and higher order thick plate theories.

DISCUSSIONS ON RESULTS FOR BUCKLING COEFFICIENT (λ)

In connection with the results for buckling coefficient (λ) presented in Table 1, the following observations can be noted:

NOMENCLATURE

a	Length of the plate	N_{cr}	Critical buckling load
b	Width of the plate	Q_x, Q_y	Shear forces
D	Plate rigidity	u, v, w	Displacements along x, y and z-directions, respectively
E	Young's or Elastic modulus		
G	Elastic modulus in shear	$q(x,y)$	Intensity of transverse load
h	Thickness of the plate	q_o	Uniform transverse load
M_x, M_y	Bending moments	μ	Poisson's ratio
N_x, N_y	In-plane loads	∇^2	Laplace operator
N_0	In-plane load of uniform intensity	λ	Buckling coefficient

Buckling coefficients predicted by SVSPT for the case of simply supported square plates are accurate in comparison with exact theory or theory of elasticity results for buckling loads. The buckling loads obtained by SVSPT are more or less same as those predicted by other thick plate theories, namely, Mindlin, Reddy and RPT plate theories cited in Table 1.

Only CPT results differ as the plate thickness is increased. This is in well agreement with the discussions presented in the previous sections that, CPT can only be used only for the investigation of thin plates as it neglects the shear deformation effects.

5.0 Conclusions

In this paper, the SVSPT could be used successfully and in a simplistic manner for the buckling study of thick plates with simply supported edge conditions. The plate analysis using SVSPT is simpler as the formulation leads to only one governing differential equation. Also, as the governing equation of SVSPT is closely similar to that of CPT, the plate analysis can be carried out in the similar lines of CPT. In case of SVSPT, all the expressions associated with plates can be expressed in terms of a single unknown displacement variable. This reduces the complexity in the plate analysis. The efforts involved in solving plate problems using SVSPT is considerably less when compared to the plate analysis using other higher order thick plate theories. The buckling loads predicted by SVSPT are in good agreement with the results predicted by other thick plate theories.

References

[1] S. Timoshenko and S. Woinowsky-Krieger, (1959): "Theory of Plates and Shells", 2nd edition, McGraw-Hill Book Company, New York.

[2] S. P. Timoshenko and G. M. Gere, (1961): "Theory of Elastic Stability", 2nd edition, McGraw-Hill Book Company, New York.

[3] R. D. Mindlin, (1951): "Influence of rotary inertia and shear on flexural motions of isotropic elastic plates", *ASME Journal of Applied Mechanics*, Vol.18, pp.31-38.

[4] J. N. Reddy, (1984): "A Simple Higher-order Theory for Laminated Composite Plates", ASME *Journal of Applied Mechanics*, Vol. 51, No. 4, pp. 745–752.

[5] J. N. Reddy, (1984): "A refined nonlinear theory of plates with transverse shear deformation", *International Journal of Solids and Structers*, Vol. 20, No. 9, pp. 881–896.

[6] R. P. Shimpi, (2002): "Refined plate theory and its variants", *AIAA Journal*, Vol. 40, No. 1,, pp. 137–146.

[7] C. M. Wang, J. N. Reddy and K. H. Lee, (2000): "Shear Deformable Beams and Plates: Relationships With Classical Solutions", 1st edition, Elsevier Science Ltd., Amsterdam.

[8] C. M. Wang and W. A. M. Alwis, (1995): "Simply supported polygonal Mindlin plate deflections using Kirchhoff plates", *Journal of Engineering Mechanics*, Vol. 121, No. 12, pp. 1383–1385.

[9] C. M. Wang, G. T. Lim and K. H. Lee, (1999): "Relationships between Kirchhoff and Mindlin bending solutions for Levy plates", *ASME Journal of Applied Mechanics*, Vol. 66, pp. 541–545.

[10] C. M. Wang, G. T. Lim, J. N. Reddy and K. H. Lee, (2001): "Relationships between bending solutions of Reissner and Mindlin plate theories", *Engineering Structers*. Vol. 23, No. 7, pp. 838–849.

[11] K. H. Lee, G. T. Lim and C. M. Wang, (2002): "Thick Levy plates re-visited", *International Journal of Solids and Structers*, Vol. 39, No. 1, pp. 127–144.

[12] J. N. Reddy and N. D. Phan, (1985): "Stability and vibration of isotropic, orthotropic and laminated plates according to a higher-order shear deformation theory", *Journal of Sound and Vibration*, Vol. 98, No. 2, pp. 157–170.

[13] J. N. Reddy and C. M. Wang, (1998): "Deflection relationships between classical and third-order plate theories", *Acta Mechanica*, Vol.130, No.3-4, pp. 199–208.

[14] I. Shufrin and M. Eisenberger, (2005): "Stability and vibration of shear deformable plates–first order and higher order analyses", *International Journal of Solids and Structures*, Vol. 42, No.3, pp. 1225–1251.

[15] S. H. Hashemi, M. Fadaee and H. R. D. Taher, (2011):

"Exact solutions for free flexural vibration of Levy-type rectangular thick plates via third-order shear deformation plate theory", *Applied Mathematical Modelling*, Vol. 35, No. 2, pp. 708-727.

[16] H. T. Thai and D. H. Choi, (2013): "Analytical solutions of refined plate theory for bending, buckling and vibration analyses of thick plates", *Applied Mathematical Modelling*, Vol. 37, No.18, pp. 8310–8323.

[17] H. T. Thai and S. E. Kim, (2012): "Analytical solution of a two variable refined plate theory for bending analysis of orthotropic Levy-type plates", *International Journal of Mechanics*, Vol. 54, No. 1, pp. 269–276.

[18] H. T. Thai and S. E. Kim, (2012): "Levy-type solution for free vibration analysis of orthotropic plates based on two variable refined plate theory", *Applied Mathematical Modeling,* Vol. 36, No. 8, pp. 3870–3882.

[19] R. P. Shimpi and H. G. Patel, (2006): "Free vibrations of plate using two variable refined plate theory", *Journal of Sound and Vibration*, Vol. 296, No.4, pp. 979–999.

[20] R. P. Shimpi and H. G. Patel, (2006): "A two variable refined plate theory for orthotropic plate analysis", *International Journal of Solids and Strucures*, Vol. 43, No. 22, pp. 6783–6799.

[21] R. P. Shimpi, R. A. Shetty and A. Guha, (2017): "A single variable refined theory for free vibrations of a plate using inertia related terms in displacements", *European Journal of Mechanics*-A/Solids, Vol. 65, pp. 136-148.

[22] S. Srinivas, and A. K. Rao, (1969): "Buckling of thick rectangular plates", *AIAA Journal*, Vol. 7, No. 8, pp. 1645–1646.

[23] S. H. Hashemi, K. Khorshidi and M. Amabili, (2008): "Exact solution for linear buckling of rectangular Mindlin plates", *Journal of Sound and Vibration*, Vol. 315, No. 1, pp. 318–342.

[24] J. N. Reddy and N. D. Phan, (1985): "Stability and vibration of isotropic, orthotropic and laminated plates according to a higher-order shear deformation theory", *Journal of Sound and Vibration*, Vol. 98, No. 2, pp. 158–170.

[25] Venkatesha B K, Suresh B S, and Girish K E, (2012): Analytical Evaluation of Fatigue Crack Arrest Capability of Fuselage in Large Transport Aircraft, International *Journal on Theoretical & Applied Research in Mechanical Engineering,* ISSN: 2319-3182, 1(1), pp.13-22.

[26] Venkatesha B K, Prashanth K P, and Deepak Kumar T, (2014): Investigation of Fatigue Crack Growth Rate in Fuselage of Large Transport Aircraft using FEA Approach, *Global Journal of Research in Engineering*-USA, ISSN: 2249-4596, 14(1), pp.11-19.