70(8A): 1-479; 2022. DOI: 10.18311/jmmf/2022/32013

Print ISSN : 0022-2755
Journal of Mines, Metals and Fuels
Contents available at: www.informaticsjournals.com/index.php/jmmf

A Review on Non-Linear and Mathematical Modelling for Stiffness Characteristics of Linear Motion Guide Ways

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Abstract:

Linear motion guide ways moves along a predetermined path. Linear Motion guide way features High Positioning, Repeatability, Low Frictional Resistance. Linear Motion guide ways finds wide applications in Aerospace, Automotive, Medical, Pharmaceutical, Machining Tools, Industrial Robots. The stiffness of a Linear Motion guide way is important for the static & dynamic behaviours of machine. The worldwide work focuses stiffness from load and deflection point which in turns help the designers and Machine tool manufacturers to build guide ways for suitable applications. Some studies reveals that the load (Vertical and Horizontal) and deflection in guide ways are having non-linear behaviour. Less work has been done till date considering the non-linear behaviour. Major studies have focussed on Vertical, while few have nominally considered horizontal loading. We wish to present a study considering effect of Vertical, Horizontal and Combined loading on Linear Motion guide ways. We also propose to develop a mathematical model to optimize the stiffness values for Linear Motion guide ways having different rolling elements. Studies done so far reveal clearly that this nature of study has not been undertaken till date and we expect that our work will benefit Research Scholars, Academic Institutes along with designers and Machine tool Manufacturers.

Keywords: *Linear Motion Guideways, Rolling Element, Stiffness, Combined Stiffness, Non- Linear Behaviour, Mathematical Model*

1.0 Introduction

In the paper the work carried out so far in regards of stiffness characteristics of linear motion guideways is discussed. The methodology adopted by various researchers has been presented and the work that can be carried out for non-linear stiffness characteristics and developing of mathematical model has been proposed. The work will definitely benefit designers and machine builders from designing and manufacturing aspects.

1.1 Review of Literature

Discusses theoretical relationship for linearly moving ball guides depending upon a three-parameter life equation based on basic dynamic load has been derived. Stress cycles accumulated on guide and rails, rolling elements and elements equivalent average load are calculated to include the crowning effect on guides. The values for basic dynamic load are then computed for different strokes, load eccentricity and fitting factors.

Describes about machine tool properties depend greatly on the rigidity of its construction system and includes the stiffness of all system components. In load bearing systems of machine stiffness is important for existing fixed and movable joints. Finite element analysis method is useful to calculate static properties of path joints. The phenomena that occurs in the contact layers of respective junctions is considered in the presented method of computational methods.

Analysis rigid shaft supported by Ball bearings subjected to axial and radial vibrations. Non-linear spring contacts are considered in between races and balls for analytical formulations. Hertzian elastic contact deformation theory obtains stiffness required. Lagrange's equation helps to obtain governing differential equations. To solve the non-linear differential equations implicit type numerical integration technique is used. Number of balls and preload parameters are investigated in perfect bearings.

Explains machine elements like balls and roller bearings are widely used in motion guidance of machine tools exhibits frictional hysteresis. In order to achieve good design and efficient control on guidance of machine tools it is imperative to study the non-linear dynamics caused by the elements involved. Paper puts forth the concept of equivalent stiffness and damping in linear systems having rolling interfaces exhibiting hysteresis friction. Paper's work is much devoted to damping ratio experimental correlation with rolling elements bearing design parameters.

Examines Equation of stiffness for linear guide recirculation rollers having arbitrary crowned profiles has been derived in the paper. Simulation is done as rollers compressed between two plates with a block and is divided in three parts, in actual rollers are compressed in between profiled track and a block. Rollers with circular, cubic, quadratic, exponential and quaternary power profiles are analyzed using superposition method to obtain stiffness equation for rollers. To obtain the equation of stiffness for linear motion recirculation rollers a discrete normal spring model is used.

The article describes vertical stiffness in preloaded linear motion guideways and incorporates flexibility of the rail and carriage. Vertical loads are applied on light and medium preloaded bearings and deformation of carriage is measured for obtaining the stiffness. Comparison revealed that stiffness for conventional rigid model was 40% lesser. Vertical loading conditions resulted in side faces deformation of carriage. Results revealed that minimal deformation occurred at the top, increased trend at bottom also it had a greater tendency under either a vertical load that is small or more preload. It is not possible to explain the measured stiffness and external deformation of the carriage using a conventional stiffness model. Flexible model was introduced in the work to overcome the problems encountered in conventional rigid model. Finite element analysis was done to estimate the deformations in the flexible model. The results showed a good agreement for calculations and measurements done for flexible and conventional rigid model.

Examines performance related to structure for vertical milling machine that can be enhanced by focussing on the dynamic characteristics of the column spindle system. Linear guides are used to fed the spindle head through a ball screw driver. A finite element model for linear components incorporating contact stiffness at rolling interface was developed for the study to evaluate the dynamic characteristics of a column spindle under the effect of linear guides. Vibration test and finite simulation revealed the fact that preloading of guides affects greatly the vibration behaviour associated with spindle head. Increased preload on linear guides enhances the dynamic stiffness of spindle head. Derived results showed good similarities for simulations and experimental measures. The model can be applied to investigate dynamic characteristics of various configuration machine tool systems.

Paper focuses on stiffness characteristics of linear rolling ball guideways. Point contact exists between balls and the groove in guideways. Precision and rigidity of the machines is greatly influenced by stiffness. Preload is responsible for affecting the stiffness in the guideways and reduces the deviations of positions under the effect of external loading. Authors have developed a finite element model and has compared it with actual experimental results to prove the accuracy of the model. Model can be used to predict stiffness of guideways with four rolling ball rows, face to face set up and angular contact of 45 degrees for various preload. The model will prove beneficial for stiffness establishment on linear guides with different sizes and arrangements.

Discusses the key problem in modelling of complex structure is identification of joint parameters. The work aims at effectively solving joint problems by finite element analysis, while joint parameters stiffness and damping of linear motion guides is measured using frequency response functions. Experimental results were compared to hertz and frequency domain for correctness. Finite Element based model analysed the linear rolling guide's dynamic characteristics. The study can be applied to assess dynamic characteristics of various configurations mechanical system.

Reports machine tools are capable of high precision and speed machining. Machining stability prediction is very important for design of machine tools. Machine tool structure frequency characteristics and the cutting process dynamics determines the machining performance. This can be expressed in terms of stability lobe diagram. The study aims at developing finite element model for evaluating vertical milling systems machining stability and dynamic characteristics. Contact stiffness in rolling interfaces defined by hertz theory was used on linear components and machine structures in Finite Element Model. Model determined that vibration mode had dominating influence on machining stability and stiffness. The simulation results projected that linear guides with different preloads affects machining stability and dynamic behaviour of vertical column spindle head. Model proposed can be used to determine dynamic performance of machine tool system for various configurations of linear rolling elements.

Demonstrates complete models of FEA can consume huge amounts of degrees of freedom, effective steps are needed to reduce degrees of freedom consumption of rolling elements to simpler equivalent models. Guide blocks of linear guides consists of three different parts and they are firmly held together by friction and pretension. The simulation performed here has not been reported in earlier reviews, the model presents three equivalent contact models. Merits and demerits of equivalent models are addressed for full model and results for static stiffness simulations are provided. Hertz analytical solutions are used to validate the results of numerical models.

Describes machines tool dynamic analysis on joint is an important factor and it has greater effect on the machining performance of machine. Preloading is an important operation parameter and it influences greatly damping and stiffness in machine tool joints. The paper work aims at presenting the effect of preloads on dynamic stiffness of spindle nose in Horizontal Machining centres. Studies are carried out on ball screw joints, linear guides and bolts in regards of distribution and types of machine tool joints. Hertzian contact theory is used to calculate the preload influence on axial stiffness of a ball screw. Paper also discusses the pre-tightening effect on pressure of bolt joints. Optimum algorithm, Simulation and experimental results are obtained for dynamic stiffness and damping in linear guides. Machine tool structures Finite element model is built up considering different joint effects and test results are verified against it to predict the preload influence on linear guides and ball screws. Results clearly state that dynamic stiffness of spindle nose is significantly influenced by preloads in machine tool joints.

The authors have proposed an experiment to evaluate quantitative method for Damping Capacity of Linear Rolling Bearings. Low friction in Linear Rolling Bearings makes its usage often in machines. There is no evaluation method for quantitative damping. Authors have developed experiment and mathematical model for estimating stiffness and damping capacity of LM guideways.

Discusses that under the event of unbalanced bending loads the study of behavioural dynamic of ball type contact surfaces are done. Hertz contact theory based simulation is used for three dimensional finite element, point to point analysis and a model test. Best results are obtained in Finite Element Model as compared to others. The reason being number of elements can be taken up for studying it being carriage, rails, steel balls etc. The work suggests that external loads can significantly affect the dynamic properties of structures so it is of much importance to select appropriately contact stiffness in the simulations.

Comparison for theoretical and experimental studies on stiffness of linear guide rails without backlash without preload have been carried out. The work proposes to apply contact theory of Hertz's to the steel balls for deriving the equation on angle of contact. Analysis of theoretical work shows that under different external loads the value of stiffness varies non-linearly. An Experimental set up is developed wherein three blocks stiffness curves having variations in geometries and steel balls assembled are subjected to vertical loads till the guideways undergo plastic deformation. Results showed that stiffness is not affected by width and thickness. Relative error of 4.5% was revealed comparing with theoretical and experimental work. Results will be useful in deciding stiffness of linear guides in design stage resulting in determining static and dynamic behaviour.

Explains computer numerical control machines widely uses roller linear guideways. The performance and service life of guideways is greatly impacted by stiffness and wear. In order to optimize the design and enhance the performance, is important to investigate the stiffness and wear characteristics of roller linear guideways. Contact stiffness calculation model based on rigid body dynamics and palmgren formula was established. Wear analysis process of roller guides was done by using Archard wear theory. In order to predict the wear loss of roller linear guides calculation model is developed for wear loss happening in slider's raceway under loading conditions. A specialized test system is used to verify the effectiveness of proposed model against simulation and experimental works.

The work presents modelling and theoretical basis including profile guide and carriage in preloaded condition. Modelling methods are discussed taking different simplification degrees and selected results are numerically analyzed. The effectiveness of the modelling methods are evaluated.

This research work deals with some numerical investigations on the stiffness of machine tools linear guides. Finite Elements Method (FEM) is employed to carry out the simulations using ANSYS software. Experiment were carried out for study of guide stiffness in different directions such as radial direction, lateral direction and angular direction. Empirical research takes a lot of time and money, necessitating the use of numerical methods at the design stage and further studies. The results are compared to the FEM model of the guides and the experimental conducted. Numerical and experimental results are well in agreement.

States that linear rolling guideways is a complex non-linear mechanical system consisting of carriage, rail, rolling elements and other accessories. The system have large number of rolling interfaces between the elements and grooves. So it becomes important to study the static deformation and vertical stiffness of guides subjected to different loading configurations. This will be helpful to obtain the guideway structural design and mechanical analysis of CNC Machine tools. The work is sfocussed on developing modelling techniques for linear rolling guideways using finite element method, analytical and experimental work. The work will prove beneficial in providing details for effect of load and preload on the static characteristics of single ball and whole guideway system. Study may be crucial in providing a reference for precise dynamic model of linear rolling guides.

Reports that performance and accuracy are influenced by stiffness and it is the useful criterions of machine's guides. Paper work is concerned with rational studies conducting using Hertzian theory long with investigations in experimental work concerning with static stiffness of guideways. Experiments were carried out to study the linear guides static stiffness, directional and linear guideways of industrial ball bearings. For 3 levels of preload in carriages, tensile and compression directions maximum force of 10,000N is applied. Investigation in experiment and analytical are compared. Various velocities are provided to guides in order to see the movement influence on the stiffness characteristics

Reports that main property of linear guide is its dynamic behaviour. Computer Numerically Controlled machines main part is linear guide pairs. The service life of linear guide pairs can be increased by doing a detailed study on its dynamic properties. The study of concern for the work relates with comparison of stiffness in linear guides by finite element approach and experimental work. In the work done stiffness of guide pairs is calculated experimentally and the appropriateness of results is confirmed by Finite element approach. For performing the work three guide rail pairs were chosen having origination from China. The work provides an insight in establishing stiffness on linear guides having different structures.

Paper proposes a novel approach for monitoring automatically preload degradation of recalculating linear ball type of guideways. Results of both experiment and simulation revealed that natural frequency reduction corresponding to worktable yawing mode was very sensitive and linearly varied with recirculating linear ball type of guideways preload. The experimental set up has three accelerometers attached to worktable and pulse from servo motor feed drive was used to excite the worktable of machine tool. Study identified the worktable natural frequencies and respective shapes of mode using operational modal analysis. Model assurance criteria was used for extracting natural frequency corresponding to worktable yawing mode and also change in frequency was tracked amongst all the natural frequencies using operational model analysis. Due to this recirculating linear ball type of guideways preload degradation can be automatically monitored without manually exciting the worktable. In the paper the methods performance was experimentally assessed.

Research places stiffness modelling of load bearing systems in machine tool using FEM. The work proposes simplified linear stiffness model with preload dependent on equivalent contact model. The model developed has four rod element instead of balls. To facilitate the model computation time was reduced by using reconstruction and reduction methods. Experimental results where compared to the proposed models accuracy. The computation time for simple, full and equivalent models were evaluated and compared. The rigidity of the machine support system is determined by the model. The model determines the stiffness of bearing system quickly and accurately.

Authors have presented a novel model of five degrees of freedom static analysis of linear guides having subjected to external loadings. In this model to obtain roller contact loads and deflection of carriage a rigid analytical model has been developed. This is due to elastic deformation at roller rail contact and roller carriage. In the profiled rail or profiles guide rails non hertzian contact loads are considered. Contact loads caused the structural deformations of the carriage and the same was computed by finite element method. Systematically associated displacements of the carriage were derived. Rigid models estimated displacement and finite element model displacement were summed up obtained to have the total displacement of the carriage top. The validation of the proposed model was done by comparing displacement calculated with a commercial program for various loading configurations. A study on preload effect on linear guideways was also performed. Results for simulation revealed distribution of internal loads on linear guides and dependency of carriages rigidity characteristics.

Discusses that linear-motion ball guides are commonly used as machine tool supporting elements . LM guideways are crucial because these assist in selection of operating parameters and geometry also aids in stiffness maximizing and friction reduction. A optimization process for multiple objective was performed for linear ball guides using particle swarm optimization. Radial stiffness, Dynamic rating load, radial stiffness were the objectives for optimization. For stiffness and friction force calculations numerical model was proposed. ISO 14728-1:2017 was used to determine basic dynamic load rating. For optimal design parameter selection Pare-to-optimal solutions was used. Cooperative equilibrium point method was used to perform Pare to optimality for satisfying the final optimal point. The performance was good for optimized linear ball guide using simulation results. Sensitivity analysis was done for understanding the effect of parametric uncertainty in the design of optimal performance. The study was extensively done.

Reports that machine tools widely use Linear rolling guideways. Guideways affect the overall performance of machine tools. Basic characteristics of joint surface were obtained by experiment. Also for single ball joint surfaces a contact model was established. The proposed model analyzes six joint stiffness. Devises for testing of six stiffness were developed. By the use of developed testing devices stiffness of the medium preloaded are achieved. Experiment revealed approximation consistency in results.

Linear guideways subjected to periodic excitation in five directions were studied as model and non-linear dynamic analysis was performed. Dynamic model for five degrees of freedom were investigated for vibration of non-linear behaviour subject to external periodic excitation. Adopting coordinate transformation made easy piece wise derivation restoring force functions and equation of motion. Numerical results revealed fascinating happenings in preload, amplitude and harmonic frequency and mass of carriage that can be linked with vibration systems.

1.2 Review Matrix for Methods used (Table 1)

2.0 Discussion

2.1 Summary about the nature of work executed by researchers

Researchers have studied the preload and its effect on the performance characteristics of linear guideways as shown in Fig.1. Preload increases the stiffness of linear guideways and reduce the position deviation under external loads. The preload puts an restriction on deformation resulting in improved stiffness of

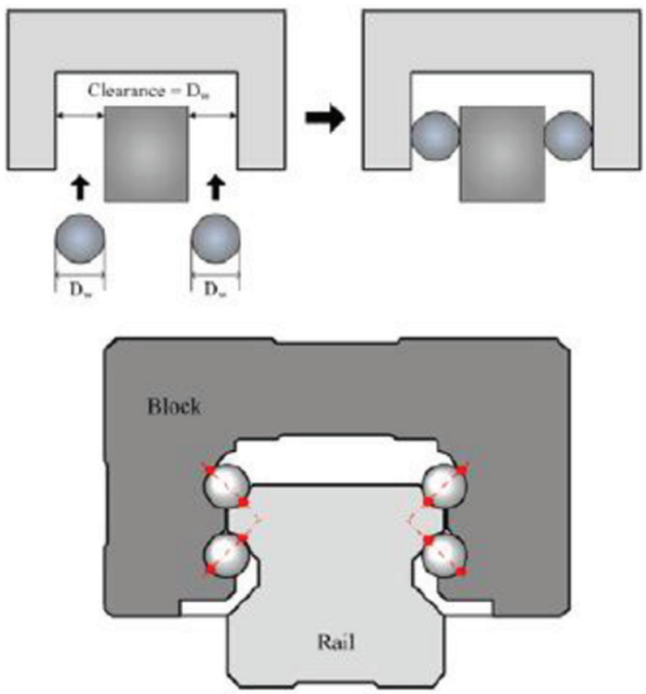

Figure 1: Zero Preload Condit ions (Court esy : D. Shaw and W. L. Su (2013))

Table 1: Review Matrix

Sr. No	Author/Authors	Year	Methods Used
1	Shimizu et al	1998	Theoreticaland Model (Equations)
2	Chlebus &Dybala	1999	FEA andComputational Methods
3	Harsha et al	2003	Experimental, Theoretical, Model
4	Al-Bender & Symens	2005	Experimental, Equation of Motion and Imperative study
5	Horng et al	2009	Simulation and Model
6	Ohta & Tanaka	2010	FEA andexperimental
7	Yuan Lin et al	2010	FE simulationand experimental
8	D Shaw	2011	Experimentaland FEA
9	L. Li & Zhang	2011	Theoretical, FEAand Experimental results
10	Hung et al	2011	FE model, Theoreticaland Experimental
11	Dadalau et al	2012	FE Simulation & Models, Theoretical and Analytical method.
12	Mi et al	2012	Optimum Algorithm, FE Simulation and Experimental results
13	Sakai & Tsutsumi	2012	Experimentand Mathematical Model
14	J. S. S. Wu etal	2012	FE Simulation, Modal test, Experimental
15	D. Shaw & Su	2013	Theoreticaland Experimental
16	Tao et al	2013	Calculationmodel, Rigidbody dynamics, Archard wear theory, Simulation and Experiments
17	Pawe³ko et al	2014	Theoreticaland modelling based
18	Mahdi et al	2015	FEM for Simulations and Experiment.
19	Sun et al	2015	Analytical, FEM and Experiment work
20	Rahmani & Bleicher	2016	Experiment and Analytical
21	Shaukharova et al	2016	FEA and Experiment
22	TSAI et al	2017	Simulation and Experimental (Operational Modal Analysis,
23	Dunaj et al	2019	FEM and Experiment work
24	Kwon et al	2019	Rigid Analytical Model, FEM
25	Tong et al	2020	Particle swarm optimization process, Numerical Model, Pare-to-optimal solutions, Cooperative equilibrium point, Sensitivity Analysis
26	J. Wang et al	2020	Experimental and Contact Model
27	M. Xu et al	2021	Non linear dynamic analysis, Coordinate transformation, Numerical Analysis

linear motion guideways. Minute changes in ball size control preloads.

Researchers have also designed and developed Experimental set ups for testing the linear motion guideways subjected to various loading conditions as shown in Fig.2. The experiments performed are compared to theoretical results for deflection and load as shown in Fig.3 can be used for calculation of stiffness.

The validation of the calculated and measured values are performed by building three dimensional models of guideways and then selecting parameters such as material, constraints on degree of freedom, meshing and then applying the loading conditions as shown in Fig.4. For the purpose of analysis the model undergoes three processes namely pre-processing, processing and post processing. The results obtained showed percentage errors or good agreements for the work done as shown in Fig.5.

The simulation/computational work required to

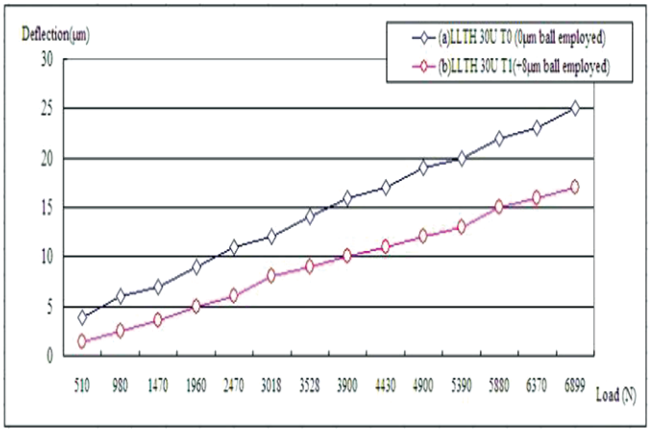

Figure 2: Effect of Zero and Preload Condit ions plott ed for Load vs. Deflect ion (Courtesy : D. Shaw and W. L. Su (2013)

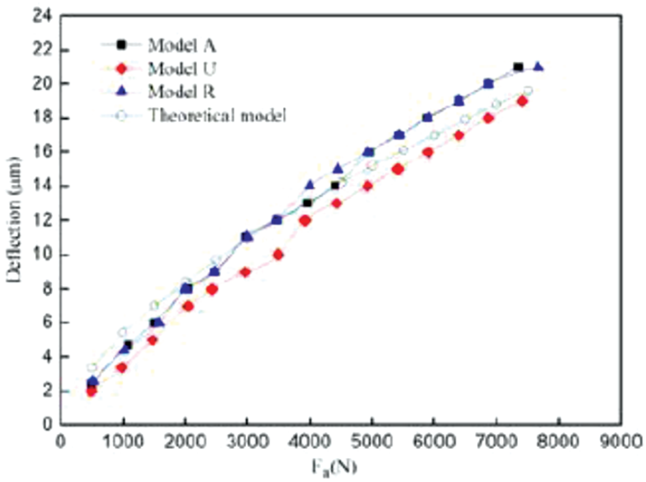

Figure 3: Linear Mot ion guide ways Load vs Deflect ion (Court esy : D. Shaw and W. L. Su (2013)

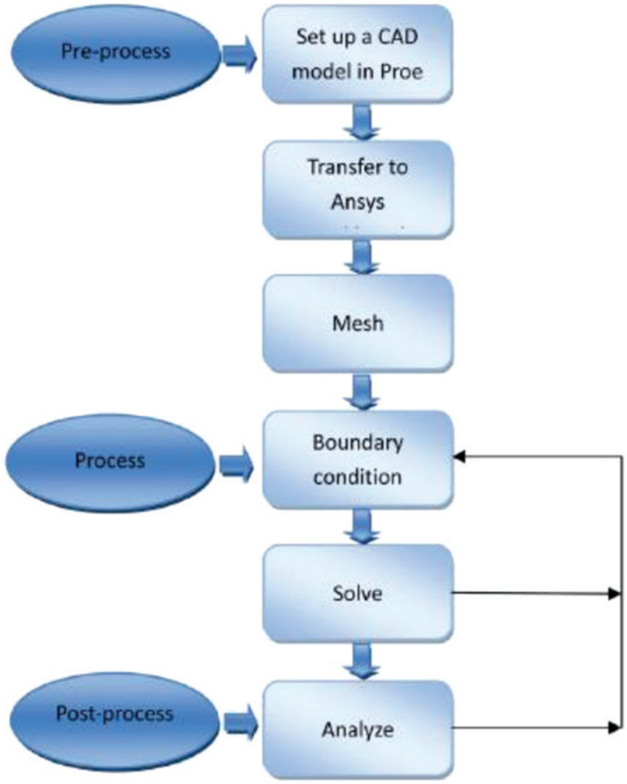

Figure 4: Flow chart of FEA process (Court esy : Azhar Shaukharova, Yi Liang, Hut ian Feng, Bin Xu (2016))

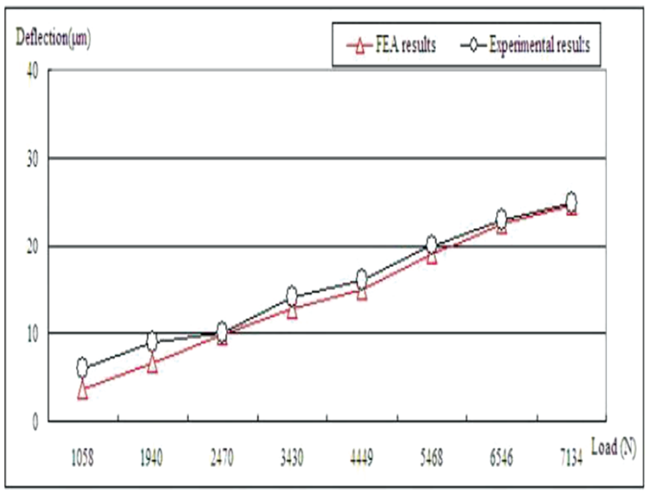

Figure 5: Results Compared for Experimental and FEA Analysis (Court esy: D. Shaw and W. L. Su (2011))

perform analysis was time consuming but researchers have came up with a method to perform it within less time and the same has been shown in Fig.6.

Few of the researchers have also undertaken experimental work and the validation of the work was done by proposing and developing mathematical model of guideways from stiffness point of view as shown in Fig.7.

Researchers are also executing work in the field of Non-linear Analysis and the results obtained can be applied to the existing theories of guideways and can also contribute to the new developments that can be undertaken in coming future.

Matrix indicates clearly about the work undertaken in Hertz theory of contact for theoretical filed, Experimental set up, Building of Models for performing Finite Element Simulations and to some extent covering few aspects related with equation of motions and developing mathematical models.

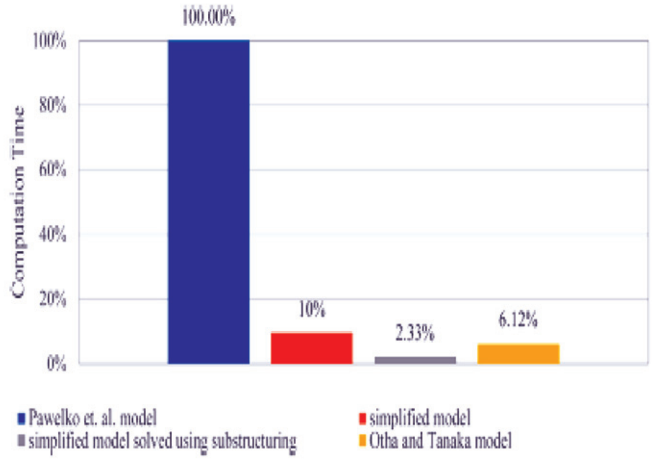

Figure 6: Comparison of Calculation time for individual models (Courtesy: Pawe³ Dunaj, Stefan Berczyñski, Piotr Pawe³ko, Zenon Grz¹dziel, Marcin ChodŸko (2019))

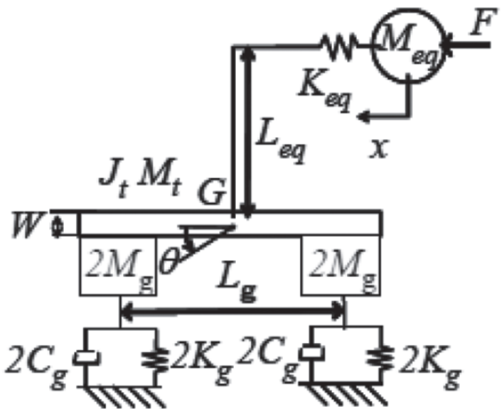

Figure 7: Mathemat ical Model (Courtesy : Yasunori SAKAI, Masaomi T SUT SUMI (2012))

3.0 Conlcusion

Based upon the literature reviewed following observations can be drawn:

The review taken highlights and strongly supports work that can be extended to perform study of Non-linear behaviour of stiffness considering vertical, horizontal and combined loads along with development of Mathematical model for optimum values of stiffness in linear guideways. In our work we will study preload, stiffness, and non-linearity aspects in much depths. Stiffness of linear motion guideway is the major factor which affects the rigidity and precision of machines and is also the key parameter for the static and dynamic behaviours of a machine. Designers and Machine manufacturers build guideways for suitable applications considering stiffness from load and deflection point. Very less work has been done taking into consideration the non-linearity aspect. All the happenings in actual are non-linear. Therefore we propose to check the non-linear behaviour exhibited by linear motion guideways in regards with Stiffness characteristics. The designers and machine builders have focused much on positioning and machining applications. Due to increasing demands of industries to opt for higher speeds, accuracy and that to at lower cost many issues are now resurfacing. So it's of great importance to provide a mathematical model which delivers an optimized solution in regards of guideways stiffness. Studies reveals clearly that this nature of work has not been undertaken till date and we expect that our work will benefit Research Scholars, Academic Institutes along with designers and Machine tool Manufacturers.

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