

# Numerical Solution of Micropolar Fluid for Jenkins Model with Micro-Rotation between Two Rotating Disks

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## Abstract

The flow of a steady, axi-symmetric, incompressible micropolar fluid between two infinite rotating disks is described for Jenkins Model. The governing equations are reduced to non-linear ordinary differential equations and are solved numerically through the shooting technique. The graphs are plotted and the impact of the material constant is analysed on the velocity, micro-rotation velocity profiles and pressure. The results reveal that the material constant has a significant effect on radial velocity, axial velocity and pressure.

**Keywords:** Ferrofluid, Material Constant, Micropolar Fluid, Rotating Disk

## 1.0 Introduction

Micropolar fluid represents fluids consisting of microelements (material particles) suspended in a medium. Microfluids are analysed through the concept of microcontinuum, in which the properties and behaviour of the fluid are affected by the material particle motions, which possess local inertia. Eringen<sup>1</sup> was the first to introduce the concept of micropolarity through microcontinuum mechanics. Eringen's microcontinuum mechanics approach to micropolar fluid led to the introduction of new kinematics variables which involve gyration tensor, micro inertia moment tensor, body moments and micro stress averages which lead to twenty-two viscosity and material coefficients. But for real, non-trivial flow problems the governing equations of a system resulted in nineteen equations in nineteen unknowns which is not easily docile to solve. Further in 1966, Eringen<sup>2</sup> considered the subclasses of these

fluids, in which he showed that in addition to a micro isotropy and if the skew-symmetric property of the gyration tensor is imposed the nineteen equations with nineteen unknowns reduces to seven equations in seven unknowns. Thus Eringen's microcontinuum approach differs from a continuum approach to a fluid. Micropolar fluids are of great consideration for research in industry and engineering. The concept of microcontinuum theory is needed to explain the many fluids, for instance flow of biological fluids in thin vessels (animal blood), polymeric suspensions, colloidal suspensions, liquid crystals etc.

Hayat *et al.*,<sup>3</sup> analysed the effect of the micropolar parameter which has a decreasing influence on the velocity profiles in radial direction and axial direction and an opposite influence on the flow during suction/injection in the study of squeezing magnetohydrodynamic flow between two parallel disks. Igor<sup>4</sup> in the study of convection in the Trapezoidal cavity, demonstrated that the micropolar fluid rate is dependent on the governing

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parameters and location of the heater (bottom, central or top position) along the inclined wall. Anur Ishak *et al.*,<sup>5</sup> studied the incompressible micropolar fluid flow generated due to a stretching sheet in the presence of radiation effect, in which it is specified that the heat transfer rate decreases with an increase in the radiation parameter. Hayat *et al.*,<sup>6</sup> studied the effect of various parameters on the micropolar fluid in the presence of a strong microelements concentration and weak microelements concentration for the steady and unsteady flow cases.

Shit *et al.*,<sup>7</sup> examined the effect of the Hartmann number, micropolar vortex parameter, unsteadiness parameter and magnetic parameter on the axial velocity, angular velocity, boundary layer and temperature profile in the study of an MHD micropolar fluid in the presence of thermal radiation over a porous stretching sheet. Tripathy<sup>8</sup> studied that the parameters had a decelerating effect on the velocity distribution along with, in which chemical reaction parameter and inertia coefficient reduces the concentration distribution and hence reducing the thickness of boundary layer in the study on micropolar fluid with chemical reaction. Ng *et al.*,<sup>9</sup> analysed the effect on the peristaltic flow of a micropolar fluid by an induced magnetic field in an asymmetric channel. Rees *et al.*,<sup>10</sup> and Prasad<sup>11</sup> analysed the micropolar fluid flow under different geometries. Srinivasacharya *et al.*<sup>12</sup> observed that the real part and imaginary part of skin friction values decrease and increase respectively, with an increase in the frequency parameter and micro rotation and the reverse is observed in the consideration of micro polarity parameter.

Eugen Magyari<sup>13</sup> investigated that the analytical solution reveals new features in analyzing the micropolar flow over a Permeable surface. Damesh<sup>14</sup> studied micropolar fluid over a stretched permeable uniform surface in consideration of the combined effect of heat generation or absorption and chemical reaction. Qin *et al.*,<sup>15</sup> analysed that the rotation and temperature have a reverse effect on a critical Rayleigh number and concluded that the rotation with the higher values of the micropolar parameters has a stabilizing effect in the case of stationary convection whereas for the oscillatory convection, the graph of micropolar fluid and Newtonian fluid represents not much of difference.

Rosali *et al.*,<sup>16</sup> in the study of the flow of a micropolar fluid with suction in a porous medium towards a stretching/shrinking sheet analysed that there exists a dual solution in the case of a stretching sheet leading to two different velocities and temperature profile, confirming the first solutions are stable where the later does not physically exists. In the case of shrinking sheets, it leads to a unique solution. Narayana *et al.*,<sup>17</sup> and Hussain<sup>18</sup> discussed the flow of a micropolar fluid in a rotating disk. Das<sup>19</sup> discussed the effect of various parameters which have either the increasing or decreasing effect on translational velocity, micro rotation and boundary layer thickness in the study of an incompressible micropolar fluid flow in a uniform porous medium.

Khedr<sup>20</sup> examined the effect of parameters such as Prandtl number, Hartmann number, heat generation/absorption parameter and wall transfer parameter on the micro rotation, velocity and temperature profile for a micropolar fluid flow past a uniform plate. Pooya Pasha<sup>21</sup> *et al.*, applied various numerical methods to analyse the accuracy of the flow through a permeable plate of a micropolar fluid with heat transfer. Pattanaik *et al.*,<sup>22</sup> studied the micropolar fluid past a stretching sheet in the presence of a uniform magnetic field with an exponential heat source. Shahzad *et al.*,<sup>23</sup> examined the effects of various parameters such as stretching ratio parameter, stretching Reynolds number, bioconvection Rayleigh number and vortex viscosity parameter on the radial, axial and angular velocities, microrotation etc. near the lower and upper disk of the system in the study of flow between double disks of a bioconvective micropolar nanofluid. The present paper problem under the Jenkins Model is not been discussed elsewhere.

## 2.0 Formulation

We consider the flow of a steady, incompressible and axisymmetric flow of micropolar fluid between two infinite plates rotating about the  $z$ -axis. The flow is induced due to the rotation of the lower disk with a constant angular velocity  $\omega$ . The components are considered in the form of cylindrical co-ordinates  $r, \theta, z$ . Here, the flow is described based on the Jenkins Model.

The governing equations of a micropolar fluid are

$$\nabla \cdot \vec{q} = 0, \quad (1)$$

$$(\lambda + 2\mu + k)\nabla(\nabla \cdot \vec{q}) - (\mu + k)\nabla \times \nabla \times \vec{q} + k\nabla \times v - \nabla p + \rho \vec{f} + \frac{\rho\beta^2}{2}\nabla \times \left(\frac{\vec{M}}{M} \times (\nabla \times \vec{q}) \times \vec{M}\right) = \rho \dot{\vec{q}} \quad (2)$$

$$(\alpha + \beta + \gamma)\nabla(\nabla \cdot v) - \gamma(\nabla \times \nabla \times v) + k\nabla \times \vec{q} - 2kv + \rho \vec{l} = \rho j \dot{v} \quad (3)$$

where  $\vec{q}$  is the velocity,  $\vec{v}$  is the micro-rotation,  $\rho$  is the density  $p$  is the pressure,  $\vec{f}$  and  $\vec{l}$  are the body force and couple stress,  $\vec{M}$  is the magnetic field,  $j$  is the micro-inertia and  $\alpha, \beta, \gamma, k, \lambda$  and  $\mu$  are material constants and dot represents the material derivative. Subject to conditions with neglected body force and couple stress of the flow the Equations (1) to (3) are written in the following form

**Equation of Continuity**

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0, \quad (4)$$

**Equation of Motion**

$$\begin{aligned} &-\frac{1}{\rho} \frac{\partial p}{\partial r} - \frac{H_0\beta^2\bar{\mu}}{2} \left[ \frac{\partial^2 v_r}{\partial z^2} - \frac{\partial^2 v_z}{\partial z \partial r} \right] \\ &+ \frac{(\mu + k)}{\rho} \left[ \frac{\partial^2 v_r}{\partial r^2} + \frac{\partial}{\partial r} \left( \frac{v_r}{r} \right) + \frac{\partial^2 v_r}{\partial z^2} \right] - \frac{k}{\rho} \frac{\partial v_\theta}{\partial z} \\ &= \left[ v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r} \right] a \end{aligned} \quad (5)$$

$$\begin{aligned} &\frac{H_0\beta^2\bar{\mu}}{2} \left[ \frac{\partial^2 v_\theta}{\partial r^2} + \frac{\partial}{\partial r} \left( \frac{v_\theta}{r} \right) \right] + \frac{(\mu + k)}{\rho} \left[ \frac{\partial}{\partial r} \left( \frac{v_\theta}{r} \right) + \frac{\partial^2 v_\theta}{\partial r^2} + \frac{\partial^2 v_\theta}{\partial z^2} \right] \\ &+ \frac{k}{\rho} \left[ \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} \right] = \left[ v_r \frac{\partial v_\theta}{\partial r} + v_z \frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r} \right], \end{aligned} \quad (6)$$

$$\begin{aligned} &-\frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{H_0\beta^2\bar{\mu}}{2} \left[ \left( \frac{\partial^2 v_r}{\partial r \partial z} - \frac{\partial^2 v_z}{\partial r^2} \right) + \frac{1}{r} \left( \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} \right) \right] \\ &+ \frac{(\mu + k)}{\rho} \left[ \frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2} \right] + \frac{k}{r} \frac{\partial}{\partial r} (rv_\theta) \\ &= \left[ v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} \right], \end{aligned} \quad (7)$$

$$\begin{aligned} &(\alpha + \beta + \gamma) \left[ \frac{\partial^2 v_r}{\partial r^2} + \frac{\partial}{\partial r} \left( \frac{v_r}{r} \right) + \frac{\partial v_\theta}{\partial r \partial z} \right] \\ &- \gamma \frac{\partial}{\partial z} \left( \frac{\partial v_\theta}{\partial r} - \frac{\partial v_r}{\partial z} \right) - k \frac{\partial v_\theta}{\partial z} - 2kv_r \\ &= \rho j \left[ v_r \frac{\partial v_r}{\partial r} - \frac{1}{r} v_\theta v_r + v_z \frac{\partial v_r}{\partial z} \right] \end{aligned} \quad (8)$$

$$\begin{aligned} &(\alpha + \beta + \gamma) \left[ \frac{\partial^2 v_\theta}{\partial z \partial r} + \left( \frac{\partial v_\theta}{r} \right) + \frac{\partial v_\theta}{\partial z^2} \right] \\ &- \gamma \left( \frac{\partial v_r}{\partial r \partial z} - \frac{\partial^2 v_\theta}{\partial r^2} \right) - \gamma \frac{1}{r} \left( \frac{\partial v_r}{\partial z} - \frac{\partial v_\theta}{\partial r} \right) + \frac{k}{r} \frac{\partial}{\partial r} (rv_\theta) - 2kv_\theta \\ &= \rho j \left( v_r \frac{\partial v_\theta}{\partial r} + v_z \frac{\partial v_\theta}{\partial z} \right) \end{aligned} \quad (9)$$

$$\begin{aligned} &\gamma \left[ \frac{\partial v_\theta}{\partial r^2} + \frac{\partial}{\partial r} \left( \frac{v_\theta}{r} \right) + \frac{\partial^2 v_z}{\partial z^2} \right] + k \left( \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} \right) - 2kv_z \\ &= \rho j \left( v_r \frac{\partial v_z}{\partial r} + \frac{1}{r} v_z v_r + v_z \frac{\partial v_z}{\partial z} \right) \end{aligned} \quad (10)$$

The boundary conditions are:

$$\begin{cases} v_r = 0, v_\theta = r\omega, v_z = 0, v_r = 0, v_\theta = 0, \\ v_z = 0, p = 0, \text{ at } z = 0; v_r = 0, v_\theta = 0, v_z \\ \text{tends to finite negative value,} \\ v_r = 0, v_\theta = 0, v_z = 0 \text{ at } z = h_1; \end{cases} \quad (11)$$

Using the dimensional analysis as mentioned<sup>24</sup>

$$\begin{cases} v_r = r\omega F_r(\eta), & v_\theta = r\omega F_\theta(\eta), & v_z = h_1\omega F_z(\eta), \\ p = -(\mu + k)\omega P(\eta), v_r = \frac{r\omega}{h_1} G_r(\eta), & v_\theta = \frac{r\omega}{h_1} G_\theta, \\ v_z = \omega G_z(\eta) \text{ where } \eta = \frac{z}{h_1} \end{cases} \quad (12)$$

where,  $\omega$  is the rotation and  $\eta$  is the dimensionless parameter of  $z$ .

The system of Equations (4) to (10) reduces to the non-linear ordinary differential equations.

$$F'_\theta + 2F_r = 0, \tag{13}$$

$$F''_r - \eta_1 G'_\theta = \eta_2 (F_r^2 + F_z F'_r - F_\theta^2) \tag{14}$$

$$F''_\theta + \eta_3 G'_r = \eta_4 (2F_r F_\theta + F'_\theta F_z) \tag{15}$$

$$+\eta_5 F'_r + 2\eta_3 G_\theta + P' = \eta_4 F_\theta F'_\theta \tag{16}$$

$$G''_r - \eta_6 (F'_\theta + 2G_r) = \eta_7 (F_z G'_r + F_r G_r - F_\theta G_\theta) \tag{17}$$

$$G''_\theta + \eta_6 (F'_r + 2G_\theta) = \eta_7 (F_r G_\theta + F_\theta G'_r + F_z G'_\theta) \tag{18}$$

$$G''_z + 2\eta_8 G'_r + 2\eta_9 (F_\theta - G_z) = \eta_{10} F_z G'_z \tag{19}$$

where the

$$\begin{aligned} \eta_1 &= \frac{k}{(\mu + k) - \bar{\beta}}, & \eta_2 &= \frac{\rho\omega h_1^2}{(\mu + k) - \bar{\beta}}, & \eta_3 &= \frac{k}{\mu + k}, \\ \eta_4 &= \frac{\rho h_1^2 \omega}{\mu + k}, & \eta_5 &= \frac{\bar{\beta}}{\mu + k}, & \bar{\beta} &= \frac{H_0 \beta^2 \bar{\mu}}{2}, & \eta_6 &= \frac{k h_1^2}{\gamma} \\ \eta_7 &= \frac{\rho \omega j h_1^2}{\gamma}, & \eta_8 &= \frac{\alpha + \beta}{\alpha + \beta + \gamma}, & \eta_9 &= \frac{k h_1^2}{\alpha + \beta + \gamma}, \\ & & \eta_{10} &= \frac{\rho \omega j h_1^2}{\alpha + \beta + \gamma} \end{aligned} \tag{20}$$

with the boundary conditions:

$$\begin{cases} F_r = 0, F_\theta = 1, F_z = 0, G_r = G_\theta = G_z = 0 \\ p = 0 \text{ at } \eta = 0; \\ F_r = 0, F_\theta = 0, G_r = G_\theta = G_z = 0 \text{ at } \eta = 1; \end{cases} \tag{21}$$

### 3.0 Method of Solution

The reduced of system non linear ordinary differential Equations (13) to (20) along with the boundary conditions (21) are solved numerically using shooting method with the implication of Runge-Kutta 4<sup>th</sup> order and Secant method. In which the higher order non linear equations are reduced to the set of first-order differential equations, the initial approximations for  $F_4, F_5, G_4, G_5$  and  $G_6$  are considered to be [0,1] and further approximations are calculated by secant method which is used as a part of shooting method, that satisfies the boundary conditions  $\eta_\infty=1$ .

$$F_r \rightarrow F_1, F_\theta \rightarrow F_2, F_z \rightarrow F_3, G_r \rightarrow G_1, G_\theta \rightarrow G_2$$

$$\text{and } G_z \rightarrow G_3. \tag{22}$$

$$F'_1 = F_4 \tag{23}$$

$$F'_2 = F_5 \tag{24}$$

$$F'_3 = -2F_1 \tag{25}$$

$$F'_4 = \eta_1 G_5 + \eta_2 [F_1^2 - F_2^2 + F_3 F_4] \tag{26}$$

$$F'_5 = \eta_4 [F_3 F_5 + 2F_1 F_2] - \eta_3 G_4 \tag{27}$$

$$P' = 2F_4 - 2\eta_5 F_4 - 2\eta_3 G_2 - 2\eta_4 F_1 F_3 \tag{28}$$

$$G'_1 = G_4 \tag{29}$$

$$G'_2 = G_5 \tag{30}$$

$$G'_3 = G_6 \tag{31}$$

**Table 1.** Values of  $\eta_1$  to  $\eta_{10}$

| Coefficients | $\eta_1$ | $\eta_2$ | $\eta_3$ | $\eta_4$ | $\eta_5$ | $\eta_6$ | $\eta_7$ | $\eta_8$ | $\eta_9$ | $\eta_{10}$ |
|--------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-------------|
| case 1       | 0.4      | 0.88     | 0.3      | 0.8      | 0.16     | 0.2      | 0.02     | 0.5      | 0.08     | 0.01        |
| case 2       | 0.50     | 1.61     | 0.45     | 1.4      | 0.1      | 0.3      | 0.03     | 0.85     | 0.12     | 0.015       |
| case 3       | 0.74     | 1.94     | 0.6      | 1.6      | 0.10     | 0.4      | 0.04     | 1.0      | 0.16     | 0.02        |
| case 4       | 0.86     | 2.27     | 0.75     | 2.33     | 0.13     | 0.5      | 0.05     | 1.41     | 2.4      | 0.025       |

$$G'_4 = \eta_6[F_5 - 2G_1] + \eta_7[F_1G_1 - F_2G_2 + F_3G_4] \tag{32}$$

$$G'_5 = \eta_6[G_2 - F_4] + \eta_7[F_1G_2 + F_2G_1 + F_3G_5] \tag{33}$$

$$G'_6 = 2\eta_8[G_3 - F_2] - 2\eta_9G_4 + \eta_{10}F_3G_6 \tag{34}$$

Substituting the table values in Equations (22) to (34) with grid size  $h = 0.0125$  and for the cases 1-4, the effects of the parameters are presented in the form of graphs.

### 4.0 Results and Discussion

The graphs are plotted to analyse the effects of the varying parameters  $\eta_1$  to  $\eta_{10}$  on velocity, micro velocity profiles

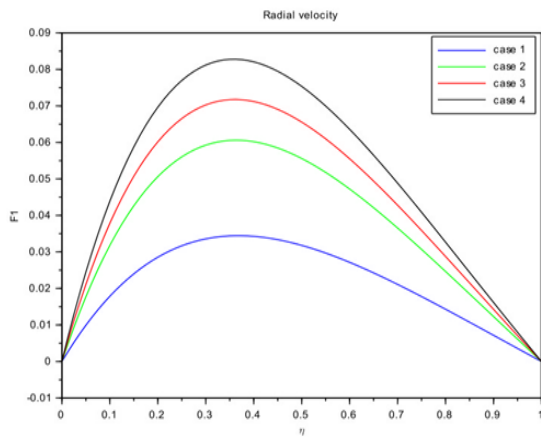


Figure 1. Radial velocity.

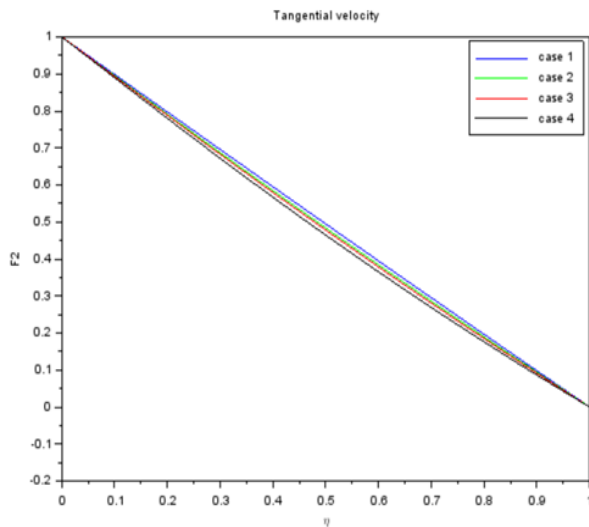


Figure 2. Tangential velocity.

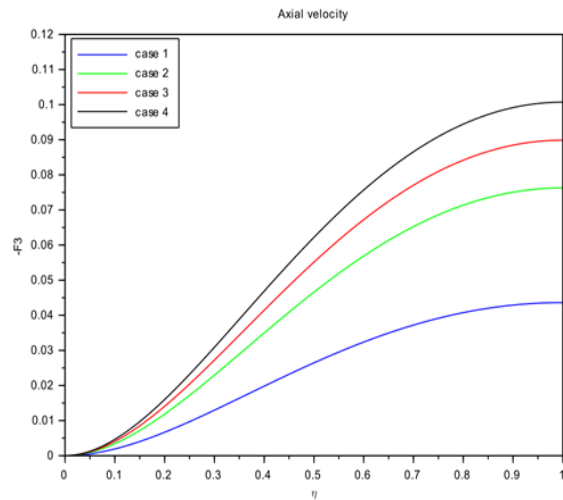


Figure 3. Axial velocity.

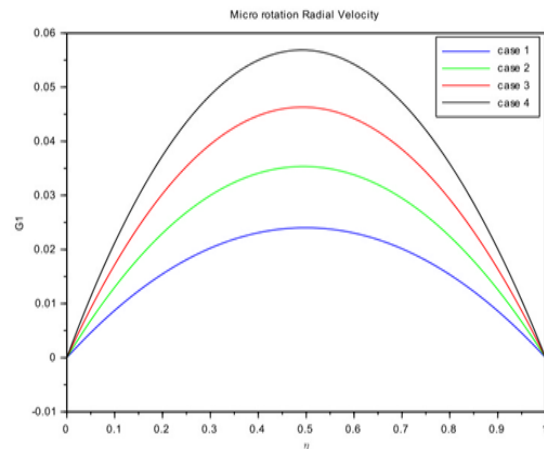
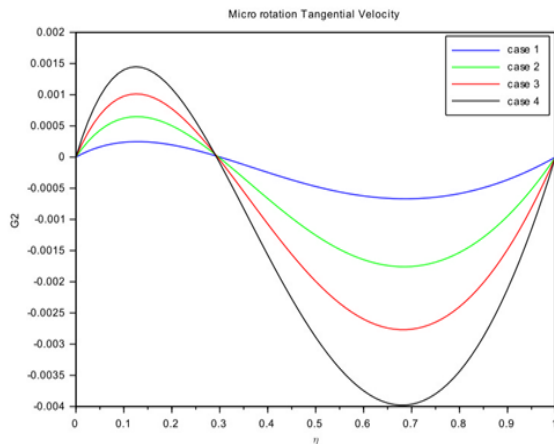


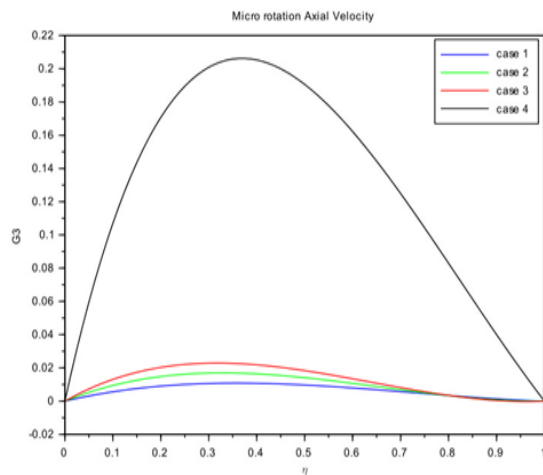
Figure 4. Radial velocity of micro rotation.

and pressure of a steady incompressible micropolar flow between constantly rotating lower and stationary upper disk. The varying values of the parameters are mentioned in Table 1. Figures 1-3 represent the velocity profiles of the fluid. Figure 1 represents the radial velocity over a dimensionless parameter  $\eta$ . From the figure it is noticed that radial velocity increases positively with an increase in the parameters stipulating the radial outward flow at the lower disk and as a maximum at  $\eta = 0.4$ , further at the upper disk approaches to zero.

Figure 2 represents the tangential velocity for varying values of parameters indicating the parameters have negligible effect on the fluid flow. Figure 3 represents the axial velocity for different values of the parameters.



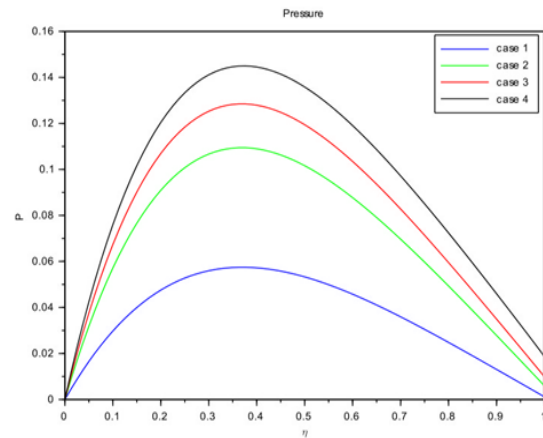
**Figure 5.** Tangential velocity of micro rotation.



**Figure 6.** Axial velocity of micro rotation.

The graph represents the negative values of the velocity specifying the inward axial flow of the fluid.

Figures 4-6 represent the micro-rotation distribution on velocity profiles over a dimensionless parameter  $\eta$ . Figures 4 and 6 represent the increasing radial velocity and axial velocity for varying values of the parameters. The graph indicates that the flow is outward and it's in the direction of the rotation of the fluid, in particular for case4 the effect of micro-rotation on axial makes a huge difference compared to other velocity profiles, as the applied magnetic field is in the z-direction. Figure 7 represents the profile of pressure for varying values of the parameters which increase with increasing values of



**Figure 7.** Pressure.

parameters which is positive both at the lower and upper disk (approaches to zero).

## 5.0 References

1. Eringen AC. Simple microfluids, *Int J Eng Sci.* 1964; 2(2):205-17. [https://doi.org/10.1016/0020-7225\(64\)90005-9](https://doi.org/10.1016/0020-7225(64)90005-9)
2. Eringen AC. Theory of micropolar fluids. *Journal of Mathematics and Mechanics.* 1966; 16(1):1-18. <https://doi.org/10.1512/iumj.1967.16.16001>
3. Hayat T, Nawaz M, Hendi AA, Asghar S. MHD squeezing flow of a micropolar fluid between parallel disks. *J Fluids Eng.* 2011; 133(11):1-10. <https://doi.org/10.1115/1.4005197>
4. Miroshnichenko IV, Sheremet MA, Pop I. Natural convection in a trapezoidal cavity filled with a micropolar fluid under the effect of a local heat source. *Int J Mech Sci.* 2017; 120:182-9. <https://doi.org/10.1016/j.ijmecsci.2016.11.028>
5. Ishak A. Thermal boundary layer flow over a stretching sheet in a micropolar fluid with radiation effect. *Meccanica.* 2009; 45(3):367-73. <https://doi.org/10.1007/s11012-009-9257-4>
6. Hayat T, Nawaz M, Obaidat S. Axisymmetric magnetohydrodynamic flow of micropolar fluid between unsteady stretching surfaces. *Appl Math Mech.* 2011; 32(3):361-74. <https://doi.org/10.1007/s10483-011-1421-8>
7. Shit GC, Haldar R, Sinha A. Unsteady flow and heat transfer of a MHD micropolar fluid over a porous

- stretching sheet in the presence of thermal radiation. *J Mech.* 2013; 29(3):559-68. <https://doi.org/10.1017/jmech.2013.33>
8. Tripathy RS, Dash GC, Mishra SR, Hoque MM. Numerical analysis of hydromagnetic micropolar fluid along a stretching sheet embedded in porous medium with non-uniform heat source and chemical reaction. *Eng Sci Technol.* 2016; 19(3):1573-81. <https://doi.org/10.1016/j.jestch.2016.05.012>
  9. Shit GC, Roy M, Ng EYK. Effect of induced magnetic field on peristaltic flow of a micropolar fluid in an asymmetric channel. *Int J Numer Method Biomed Eng.* 2010; 26(11):1380-403. <https://doi.org/10.1002/cnm.1397>
  10. Rees DAS, Pop I. Free convection boundary-layer flow of a micropolar fluid from a vertical flat plate. *IMA J Appl Math.* 1998; 61(2):179-97. <https://doi.org/10.1093/imamat/61.2.179>
  11. Prasad VR, Gaffar SA, Bég OA. Heat and mass transfer of nanofluid from horizontal cylinder to micropolar fluid. *J Thermophys Heat Transf.* 2015; 29(1):127-39. <https://doi.org/10.2514/1.T4396>
  12. Srinivasacharya D, Murthy JVR, Venugopalam D. Unsteady stokes flow of micropolar fluid between two parallel porous plates, *Int J Eng Sci.* 2001; 39(14):1557-63. [https://doi.org/10.1016/S0020-7225\(01\)00027-1](https://doi.org/10.1016/S0020-7225(01)00027-1)
  13. Magyari E, Chamkha AJ. Combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface: The full analytical solution. *Int J Therm Sci.* 2010; 49(9). <https://doi.org/10.1016/j.ijthermalsci.2010.04.007>
  14. Damseh RA, Al-Odata MQ, Chamkha AJ, Shannak BA. Combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface. *Int J Therm Sci.* 2009; 48:1658-63. <https://doi.org/10.1016/j.ijthermalsci.2008.12.018>
  15. Qin Y, Kaloni PN. A thermal instability problem in a rotating micropolar fluid, *Int J Eng Sci.* 1992; 30(9):1117-26. [https://doi.org/10.1016/0020-7225\(92\)90061-K](https://doi.org/10.1016/0020-7225(92)90061-K)
  16. Rosali H, Ishak A, Pop I. Micropolar fluid flow towards a stretching/shrinking sheet in a porous medium with suction. *Int Commun Heat Mass Transf.* 2012; 39(6):826-9. <https://doi.org/10.1016/j.icheatmasstransfer.2012.04.008>
  17. Narayana PS, Venkateswarlu B, Venkataramana S. Effects of Hall current and radiation absorption on MHD micropolar fluid in a rotating system. *Ain Shams Eng J.* 2013; 4(4):843-54. <https://doi.org/10.1016/j.asej.2013.02.002>
  18. Hussain S, Kamal MA, Ahmad F. The accelerated rotating disk in a micropolar fluid flow. *Appl Math.* 2014; 5(1):196-202. <https://doi.org/10.4236/am.2014.51020>
  19. Das K. Effect of chemical reaction and thermal radiation on heat and mass transfer flow of MHD micropolar fluid in a rotating frame of reference. *Int J Heat Mass Transf.* 2011; 54(15-16):3505-13. <https://doi.org/10.1016/j.ijheatmasstransfer.2011.03.035>
  20. Khedr ME, Chamkha MAJ, Bayomi M. MHD flow of a micropolar fluid past a stretched permeable surface with heat generation or absorption. *Nonlinear Anal-Model.* 2009; 14(1):27-40. <https://doi.org/10.15388/NA.2009.14.1.14528>
  21. Pasha P, Mirzaei S, Zarinfar M. Application of numerical methods in micropolar fluid flow and heat transfer in permeable plates. *Alex Eng J.* 2022; 61(4):2663-72. <https://doi.org/10.1016/j.aej.2021.08.040>
  22. Pattnaik PK, Mishra SR, Mahanthesh B, Gireesha BJ, Rahimi-Gorji M. Heat transport of nano-micropolar fluid with an exponential heat source on a convectively heated elongated plate using numerical computation. *Multidiscip Model Mater Struct.* 2020; 16(5):1295-312. <https://doi.org/10.1108/MMMS-12-2018-0222>
  23. Shahzad A, Imran M, Tahir M, Khan SA, Akgül A, Abdullaev S, *et al.* Brownian motion and thermophoretic diffusion impact on Darcy-Forchheimer flow of bioconvective micropolar nanofluid between double disks with Cattaneo-Christov heat flux. *Alex Eng J.* 2023; 62:1-15. <https://doi.org/10.1016/j.aej.2022.07.023>
  24. Karman V. Über laminare and turbulente Reibung. *ZAMM Z für Angew Math Mech.* 1921; 1(4):232-52. <https://doi.org/10.1002/zamm.19210010401>