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Development of Fuzzy-AHP and WPM based MCDM Model for Mining Method Selection

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Abstract

The study aims to select the proper mining method for underground metal mines using the Fuzzy-Analytical Hierarchical Process (FAHP) and Weighted Product Method (WPM). The study considered various deposit characteristics such as dip of the orebody, orebody thickness, shape of the orebody, depth of the orebody from the surface, and grade distribution. Based on these characteristics block caving, sublevel stoping, sublevel caving, room and pillar, shrinkage stoping, cut and fill stoping methods are evaluated. FAHP method calculates the weights for each deposit characteristic and WPM calculates the scores of mining methods for the given mine body characteristic. FAHP and WPM models are applied to a uranium mine in India. The appropriate extraction method for the considered mine is found as cut and fill stoping. The same method is also used at the mining location.

Keywords: *Deposit Characteristics, Fuzzy-AHP, Metal Mining Methods, Mining Method Selection, Underground Metal Mining, WPM*

1.0 Introduction

Underground mining is a tedious task in the mining industry to take the minerals out below the earth's surface without taking out the overburden. Various underground mining methods are available for mineral extraction based on the ore deposit characteristics. Block caving, shrinkage stoping, room and pillar mining, sublevel stoping, cut and fill stoping and sublevel caving etc are such extraction methods. The appropriate mineral extraction method selection depends on various deposit characteristics such as the dip of the deposit, the depth of the deposit, ore grade distribution, mining cost, etc. The deposits with flat beds of limited thickness can be extracted using room and pillar method[1]. For the deposits of massive or tabular shape and ore bodies with steep dipping can be extracted using sublevel open stoping method[2]. Cut and fill stoping can be used for the extraction of ore deposits with any shape. Deposits with tabular shapes and having steep dips can be extracted using the block caving method[3]. Shrinkage stoping can be used for mineral extraction where the dip of the deposit is steep and with stable hanging wall and foot wall[1].

The underground mining method selection problem is a Multi-Criteria Decision-Making (MCDM) type problem because the mining problem considers various deposit characteristics to evaluate proper mining methods. MCDM problem can be solved by using various decision-making approaches like Simple Additive Weighting (SAW), Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE), Analytical Hierarchy Process (AHP), Weighted Product Method (WPM), Elimination et Choix Traduisant la Realite (ELECTRE), and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS).

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In 2012, Gupta and Kumar proposed an AHP model for evaluating various mining methods for underground based on various geological conditions of the ore deposit[4]. In 2013, Ataei *et al.* designed a Monte Carlo AHP (MAHP) model for choosing the best method for bauxite deposit in Iran[5]. In 2015, Yavuz proposed AHP with Yager's technique for selecting the proper coal mining method for underground coal extraction from the deposit located in Turkey[6]. In 2017, Dehghani *et al.* used Grey with TODIM for proper mining method selection[7]. In 2018, Balusa and Gorai proposed a fuzzy-AHP technique for taking the appropriate metal mining method for any underground metal ore deposits[8]. In 2018, Balusa and Singam compared WPM with the PROMETHEE approach for underground mining method choice[9]. In 2021, Balusa and Gorai developed a hierarchical FAHP technique for mining method selection for underground mines[10]. In 2021, Mijalkovski *et al.* compared ELECTRE, PROMETHEE, AHP, and integrated AHP-PROMETHEE for choosing the appropriate mining method for underground[11]. In 2021, Balusa and Gorai applied a fuzzy pattern recognition approach for choosing the mining method[12]. In 2022, Palanikkumar *et al.* developed a fuzzy logic-based model for mining method selection for underground[13].

In previous studies, authors considered many deposit characteristics for the selection of mining methods for underground. In the present study, main deposit characteristics such as dip of ore deposit, ore body shape, depth of the ore body from the ground surface, ore deposit thickness, and grade distribution of ore are considered for mining method selection. Considering the main characteristics takes less time in evaluating the mining methods so that the best can be chosen quickly. These deposit characteristics are uncertain in reality, hence fuzzy logic-based AHP is used for weights computation. The suitable mining method can be selected after evaluating them using the WPM method.

2.0 Materials and Methods

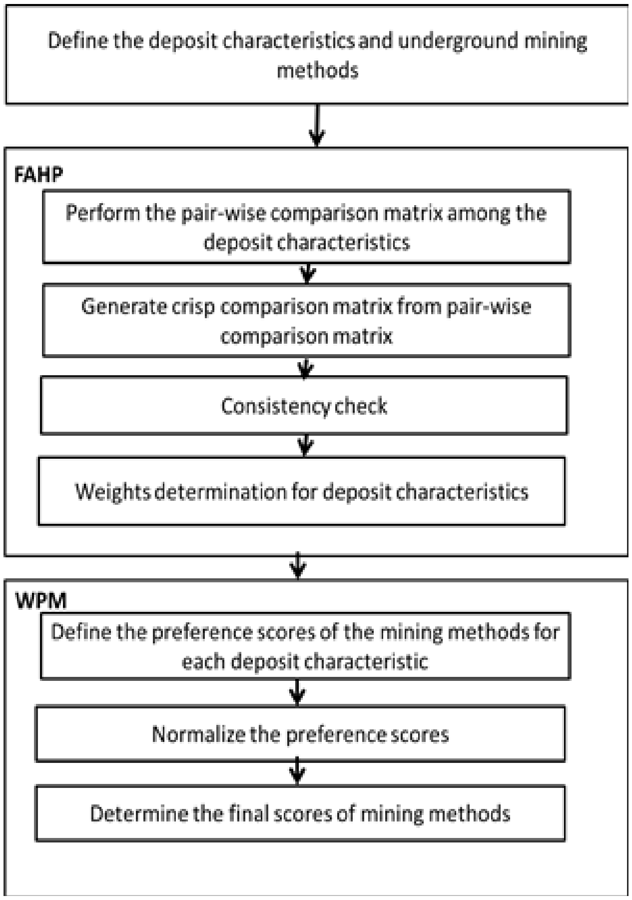

Figure 1. Methodology for choosing the right mining method for underground mines.

The methodology followed for choosing the proper metal mining method for underground in this study is shown in the flowchart in Figure 1. As mentioned in the flowchart first step is defining the deposit characteristics and underground mining methods to be considered. The second step is to use the FAHP approach to determine the weights of the deposit characteristics. The final step is to determine the proper underground mining method using the WPM approach.

2.1 Defining Deposit Characteristics and Underground Mining Methods

In the present study, deposit characteristics considered are dip of the deposit (Dp), orebody shape (Sp), thickness of the orebody (Th), orebody depth from the ground surface (De), and grade distribution of ore (Gr). The various underground mining methods considered are block caving mining (Bc), sublevel stoping mining (Ss), sublevel caving mining (Sc), room and pillar mining (Rp), shrinkage stoping mining (Sh), and cut and fill stoping mining (Cf).

2.2 Fuzzy Analytic Hierarchy Process

FAHP can be used for evaluating multiple alternatives based on multiple criteria while considering the decision maker's preference uncertainty. In the present study, using FAHP weights of the deposit characteristics are calculated. The pair-wise comparison matrix built with FAHP scale of to [14] to calculate the weights of the deposit characteristics as shown in Table 1.

2.2.1 Fuzzy Pair-Wise Comparison Matrix

A fuzzy pair-wise comparison matrix can be defined based on a comparison matrix constructed using the AHP scale shown in Table 1. Eq. (1) can be used to convert pair-wise matrix to fuzzy pair-wise comparison matrix[15]. In Eq. (1), α defines the uncertainty of the relative value considered. A fuzzy pair-wise comparison matrix defines the lower and upper bound of the fuzzy relative importance value of a pair-wise comparison matrix.

$$\bar{x}_\alpha = [x-\alpha, x+\alpha]; \frac{1}{\bar{x}_\alpha} = [\frac{1}{x+\alpha}, \frac{1}{x-\alpha}] \quad (1)$$

In Eq. (1), α value varies from 0 to 1 and in this study the value of α is assumed as 1. The α value varying from 0 to 1 represents the least confidence to the most confidence of the decision maker's value.

2.2.2 Crisp Pair-Wise Comparison Matrix

The weights of the parameters can't be calculated using a fuzzy comparison matrix. Hence, the fuzzy comparison matrix needs to be converted into a crisp comparison matrix. The fuzzy comparison matrix defined in Section 2.2.1, can be converted into a crisp pair-wise comparison matrix[16] using Eq (2).

$$a_{ij}^{\alpha} = \lambda a_{iju}^{\alpha} + (1-\lambda) a_{ijl}^{\alpha} \quad (2)$$

In the Eq. (2), a_{iju}^{α} and a_{ijl}^{α} are the upper bound and lower bound of fuzzy pair-wise comparison relative importance value a_{ij}. The defuzzified aij provides the crisp relative value. Whereas λ denotes the decision maker's attitude. In general, λ value varies from 0 and 1. The study assumed that λ as 0.5 for considering the decision maker's unbiased decision-making.

2.2.3 Consistency Check

The crisp comparison values can be inconsistent because of the preference score of the decision maker. Hence, it is required to find the consistency of a crisp comparison matrix. This can be solved by using the Consistency Ratio (CR) developed by Saaty[14]. CR for any given pair-wise comparison matrix is calculated using Eq. (3).

Table 1. FAHP scale for constructing pair-wise comparison matrix

Relative Value	**Fuzzy Scale with lower and upper bound**	**Meaning**
$\bar{1}$	(1,1,1)	Two parameters have the same weight
$\bar{3}$	(3-α), 3, (3+α)	The importance of one parameter is weaker over the other parameter
$\bar{5}$	(5-α), 5, (5+α)	The importance of one parameter is stronger over other parameter
$\bar{7}$	(7-α), 7, (7+α)	One parameter importance is demonstrated over the other parameter
$\bar{9}$	(9-α), 9, (9+α)	One parameter importance is extreme over other parameter
$\bar{2}$, $\bar{4}$, $\bar{6}$, $\bar{8}$	(x-α), x, (x+α)	Values between adjacent two relative values

Table 2. RI for different sizes of matrices

Matrices Size	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

$$CR = \frac{CI}{RI}$$

(where, CI= consistency index and RI=random index) (3)

Further, CI can be calculated using Eq. (4).

$$CI = \frac{\lambda\text{max} - \text{n}}{n - 1} \tag{4}$$

In Eq. (4), n is the crisp comparison matrix size and λ_{max} is the maximum eigenvalue of the crisp matrix.

The RI of the crisp matrix depends on the order of the matrix. The RI values for different sizes of matrices are taken from Azadeh *et al.*[17] and are given in Table 2.

2.2.4 Weights Determination for Deposit Characteristics

Many approaches have been developed for calculating the weights of parameters by various researchers. A few of these are the computation of the eigenvector, arithmetic mean, and geometric mean. This study considered the geometric mean method for weight calculation of parameters.

The geometric mean of ith row (GM_i) of a comparison matrix can be determined as:

$$GM_i = [\prod_{j=1}^{M} b_{ij}]^{1/M} \tag{5}$$

where, denotes the relative value in the crisp matrix for the i^{th} column and j^{th} row and M denotes the number of elements in the row.

The weight of the i^{th} parameter is obtained by

$$w_i = GM_i / \sum_{i=1}^{N} GM_i \tag{6}$$

2.3 Weighted Product Method (WPM)

The first step in the WPM is to define the preference scores for the mining methods for each deposit characteristic. The next step is to normalize the preference scores. The end step is to identify the appropriate underground mining method using Eq. (7).

$$P_i = \prod_{j=1}^{N} [m_{ij}]^{w_j}; i = 1, 2, \dots\dots\dots M \tag{7}$$

In Eq. (7), m_{ij} denotes the preference score of the i^{th} row and j^{th} column and w_j is the weight of the j^{th} deposit characteristic.

3.0 Application of FAHP and WPM

In the present study, deposit characteristics of the Turamdih ore deposit of Uranium Corporation of India Limited (UCIL) are taken for identifying the method using WPM. This ore deposit characteristics are given in Table 3.

Table 3. Turamdih mine orebody characteristics

Deposit Characteristic	**Characteristic Value**
Orebody dip	40° - 60° (moderate)
Orebody Shape	Irregular
Orebody thickness	Varying between 1.5m - 40m (average: 20.75m (Intermediate)
Grade distribution of the ore	Erratic
Depth of the orebody	(moderate)

3.1 Weights Determination using FAHP

Pair-wise comparison matrix for the deposit characteristics is constructed to determine the weights. The relative importance value between any two deposit characteristics is considered from the past study by Balusa and Singam[9].

$$\begin{array}{c|ccccc} & Dp & Sp & Th & De & Gr \\ \hline Dp & 1 & 3 & 1 & 4.5 & 3 \\ Sp & 1/3 & 1 & 1/2.5 & 2.5 & 1 \\ Th & 1 & 2.5 & 1 & 4 & 3 \\ De & 1/4.5 & 1/2.5 & 1/4 & 1 & 1/3 \\ Gr & 1/3 & 1 & 1/3 & 3 & 1 \end{array}$$

This comparison matrix is converted into the following matrix form using Eq. (1) by considering the lower and upper bound of each preference score.

$$\begin{array}{c|ccccc} & Dp & Sp & Th & De & Gr \\ \hline Dp & [1] & [1,4] & [1] & [3.5,5.5] & [2,4] \\ Sp & [0.25,0.5] & [1] & [0.28,0.66] & [1.5,3.5] & [1] \\ Th & [1] & [1.5,3.5] & [1] & [3,5] & [2,4] \\ De & 0.18, 0.28] & [0.28,0.66] & [0.2,0.33] & [1] & [0.25,0.5] \\ Gr & [0.25,0.5] & [1] & [0.25,0.5] & [2,4] & [1] \end{array}$$

This fuzzy matrix is further converted into the following crisp matrix using Eq. (2).

$$\begin{array}{c|ccccc} & Dp & Sp & Th & De & Gr \\ \hline Dp & 1 & 3 & 1 & 4.5 & 3 \\ Sp & 0.37 & 1 & 0.47 & 2.5 & 1 \\ Th & 1 & 2.5 & 1 & 4 & 3 \\ De & 0.23 & 0.47 & 0.26 & 1 & 0.37 \\ Gr & 0.37 & 1 & 0.37 & 3 & 1 \end{array}$$

A consistency check is done for the crisp matrix using Eqs. (3) and (4) and found that the CR value was less than 0.1. Weights of the parameters are calculated using Eqs. (5) and (6). These are shown in Table 4.

Table 4. Parameters weight

Parameter	Weight
Dp	0.34
Sp	0.13
Th	0.32
De	0.06
Gr	0.13

3.2 Ranking Mining Methods using WPM

Preference values for each mining method for Turamdih mine orebody characteristics (Table 3) are given in Table 5.

Table 5. Preference score

	Dp	Sp	Th	De	Gr
Bc	5	1	1	7	5
Ss	3	3	7	9	7
Sc	3	3	1	5	5
Rp	1	5	3	7	1
Sh	1	5	1	7	5
Cf	7	9	9	7	9

There are various ways of normalizing the data, such as Min-Max normalization, Z-score normalization, decimal scaling and logarithmic transformation. The most commonly used normalization technique is Min-Max normalization due to its simplicity. In the present study, Min-Max normalization is considered for normalizing the data. The preference scores in Table 5 are normalized and can be seen in Table 6.

Table 6. Normalized preference value

	Dp	**Sp**	**Th**	**De**	**Gr**
Bc	0.250	0.038	0.045	0.167	0.156
Ss	0.150	0.115	0.318	0.214	0.219
Sc	0.150	0.115	0.045	0.119	0.156
Rp	0.050	0.192	0.136	0.167	0.031
Sh	0.050	0.192	0.045	0.167	0.156
Cf	0.350	0.346	0.409	0.167	0.281

The appropriate extraction method is selected by calculating the preference value for each method using Eq. (7). Calculated scores and ranks for the methods are given in Table 7.

Table 7. Ranks of mining methods

	Score calculated	Rank
Bc	0.102	3
Ss	0.198	2
Sc	0.098	4
Rp	0.084	5
Sh	0.074	6
Cf	0.340	1

From Table 7, it can be seen that cut and fill stoping (Cf) results in a high score of 0.340 and the least one is shrinkage stoping having a score of 0.074. Thus, cut and fill stoping is appropriate for the extraction process for the selected orebody location. The same mining process is also used at the location.

5.0 Discussion

In this study, two MCDM models FAHP and WPM are used for the mining method selection process. Calculation of parameter weights is done by using FAHP and final score calculation of mining methods considered is done by using WPM. Deposit characteristics of the Turamdih mine are considered for validating the chosen model. Pair-wise comparison matrix between the five parameters is initialized using the FAHP 9-point scale of Saaty. In the next step, the lower and upper bound of these values are considered due to the fuzziness. This data is used for obtaining the crisp data of the fuzzy comparison matrix. A consistency check is performed for all the parameters and found that consistency lies in the standard CR value. Parameter weights are calculated using the geometric mean of the crisp comparison matrix. The preference table for all the mining methods for each parameter is initialized and normalized further. The final score value of each considered mining method is calculated using the WPM approach. These scores show that cut and cut-and-fill method is the appropriate extraction method for ore extraction at the location of Turamdih. The MCDM model proved that the result obtained and the method used at the location are the same.

Conclusions

The study attempts to develop FAHP and WPM based MCDM models for selecting the method for the Turamdih mine. In the study, five deposit parameters and six extraction processes were considered during the evaluation process. FAHP model is used for the calculation of the weight of the parameters. The WPM model is used for the calculation of scores of the methods considered in the study. The developed MCDM model is also validated with one of the Indian ore deposits. The results proved that the appropriate method found from WPM and the extraction method used at the location are the same. Hence, the considered MCDM model can also be used for any ore deposits of any location for selecting the appropriate method. As part of future work, the developed model can also validated over various deposit characteristics across the globe. The work can also be extended further by performing sensitivity analysis over various mining methods by changing the fuzzification factor (α) from 0 to 1 and decision-making attitude (λ) for 0 and 1.

6.0 Acknowledgment

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