ŚULBASŪTRAs- The source book of Vedic Geometry

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Introduction

Śulbasūtra-s form part of *Kalpasūtra*-s. *Kalpa* is one of the six *Vedāṅga*-s. The knowledge of *Vedāṅga*-s is considered essential for the complete understanding of the *veda*-s. All the *vedāṅga* texts are attributed to some author. In other words, tradition considers them as *pauruṣeya* whereas all the *veda*-s are considered as revelations (*apauruṣeya*). *Kalpa* texts are practical guides or manuals dealing with the conduct of sacrificial rituals. *Sāyaṇa* derives the word *Kalpa* as *Kalpyate samarthayate yāgaprayogo ' tra iti kalpah.* That is, *Kalpa*-s are those texts that deal with the practical conduct of sacrifices. This implies the fact that these texts presuppose the theoretical knowledge of sacrifices. In the *Samhitā*-s and *Brāhmaṇa*-s especially pertaining to the *Yajurveda,* we get mandatory rules and statements regarding procedural details of different kinds of sacrificial rituals. Moreover, *Kalpa* texts are the only systematized manuals exclusively dealing with procedural and practical aspects. The term *sūtra* denotes the style of language employed. *Sūtra* style of language is of a terse and precise kind aiming at maximum brevity. *Śulba* texts deal with the construction of different kinds of Fire altars. The word *Śulba* is derived from the root *Śulb* which means 'to measure'. Hence it means (1) the act of measuring, (2) the measure and (3) instrument used for measuring. That is how *rajju* meaning a cord came to be treated equivalent to

Śulba. *Kātyāyana*, a *Śulbakāra* begins his work with the statement *rajjusamāsam vyākhyāsyāmaḥ*.

Now, before going into the real content, nature and relative importance of the available *śulba* texts we shall have a general picture of different kinds of sacrifices. Sacrifices are broadly classified into two- *Nitya* and *Kāmya*. Based on this division, sacrificial fire itself is differentiated as *Nityāgni & Kāmyāgni*. *Nityāgni* is obligatory and *Kāmyāgni* is optional. *Nityāgni* constitutes of three fire places, *Gārhapatya, Āhavanīya* and *Dākṣiṇa*. *Dākṣiṇāgni* is sometimes called as *Anvāhāryapacana*. Every Vedic aspirant is supposed to offer daily sacrifices in *Nityāgni*. Some other seasonal obligatory sacrifices are also performed in *Nityāgni*. Based on the nature of oblations offered these sacrifices can be divided into three. They are *Iṣṭi*-s, *Paśuyajña*-s and *Somayajña*-s.

Butter and fruits are offered in the *Iṣṭi*-s. *Darśa* and *Pūrṇamāsa* are two *Iṣṭi*s. They are to be performed on New Moon and Full Moon days. *Paśuyajña*-s also called as *Niruddhapaśubandha*-s, have, as the names indicate, animals as offering. They are to be performed twice every year at the time of winter and summer solstices. Some sacrifices are to be performed, in a Vedic family, at least once in three generations. They have the *somarasa* as the offering.

There are various kinds of *Kāmyāgni*-s. They are performed for obtaining different kinds of objects or goals. They are named after the shape of the fire altar. The standard form of an optional fire altar is in the shape of a bird *śyena* (falcon). The area of this fire altar is 7½ square *Puruṣa*-s. *Puruṣa* is a unit of length (96 *aṅgula*-s,) equivalent to the measure of a man with outstretched hands. A list of various *Kāmyāgni*-s together with their shape and the desired objects of attainments as detailed in the *Taittirīya Samhitā* is given below.

Sl.No	*Agni*	Shape	Desired object
1.	*Chandaściti*		Cattle
2.	*Śyenaciti*	Falcon/Hawk	Heaven
3.	*Kaṅka*	Heron	Prominence in the world
4.	*Alaja*	A kind of bird *Pratiṣthā*,	Preeminence / High Authority
5.	*Prauga*	Isosceles triangle	Annihilation of rivals
6.	*Ubhayataḥ prauga*	Rhombus	Complete eradication of rivals
7.	*Rathacakra citi*	Chariot wheel wheels	Destruction of
8.	*Droṇa citi*	Trough	Food
9.	*Paricayyā citi*	Circle	A vast area of land / village
10.	*Śmaśāna citi*	Funeral pyre	For getting flourished in the world of manes

Now, the shape, area, and even the number of bricks to be used for the construction of all those fire altars are clearly given in the *Samhitā*-s and *Brāhmaṇa*-s of *Yajurveda*. For instance in the *Taittirīyasamhitā*, it is ordained that the *Saumikī vedi* (also called as *Mahāvedi*) should be in the form of an isosceles trapezium having its face 24 *pāda*-s, base 30 *pāda*-s and altitude 36 *pāda*-s. Similarly it is stated that *Gārhapatya citi* should be constructed with 21 bricks (*Taittirīya Samhitā* V- 2-3-5)

We also get elaborate description of rites and ceremonies in connection with the construction of these altars. Hence we can presume that the geometrical knowledge revealed in the later *Śulba*

texts was prevalent during the times of *Yajussamhitā*-s and *Brāhmaṇa*-s. S.N.Sen writes on the point : the manner in which measurements and transformations had to be carried out, in other words, the geometry and mensuration involved in their constructions, it is true, are not discussed in the *Brāhmaṇa*-s and cannot be expected either in this class of literature. But there can hardly be any doubt that what *Baudhāyana, Āpastamba* and others tried to codify in a systematic manner in their *Śulbasūtra* manuals must have for a long time formed the common property of all *adhvaryu*-s and priests specialized in the performance of sacrificial rites (The Sulbasutras, S.N.Sen and A.K.Bag, INSA, p.4)

Śulba texts

With this general introduction, we shall proceed to see the nature and content of the available *Śulbasūtra*-s. Though there are *Kalpasūtra*s attached to all the four samhitas all the known *Śulba* texts pertain to *Yajurveda*. At present we know of seven *Śulba* texts. They are *Baudhāyana, Mānava, Āpastamba, Kātyāyana, Maitrāyana, Vārāha* and *Vādhūla*. Except *Kātyāyana*, all of them belong to different recensions of Krsnayajurveda. *Kātyāyana* belongs to the *Vājasaneya* recension of *Śuklayajurveda*. (*Karavindasvamin*, a commentator of *Āpastambaśulba* refers to two other *Śulba* texts-those of *Hiraṇyakeśin* and *Masaka*. And it is interesting to note that *Masaka Śulba* belongs to *Sāmaveda*). The fact that *Yajurveda*, which is also known as Adhvaryuveda or *Yāgaveda* gave much importance to the sacrifices may be the reason behind the abundance of *Śulba* texts attached to it.

Among these seven *Śulba* texts, those of *Baudhāyana, Manu, Āpastamba* and *Kātyāyana* carry much importance. They are older

ones and they give us an insight into the early state of Hindu geometry. Bibhutibhusan Datta, the well known historian of Indian Mathematics opines that excepting *Baudhāyana, Āpastamba* and *Kātyāyana Śulba*-s all other works are influenced by the rise and advent of *Jaina* sect. The Indian National Science Academy has published all these four viz. *Baudhāyana, Āpastamba, Mānava* and *Kātyāyana Śulba*-s with English translation and elaborate English commentary in the year 1983. The editors have expressed their inability to trace out the manuscripts of other works. Regarding the date of these four *Śulba* texts, though there prevails some difference of opinion, it is generally accepted that they were written between 800 and 300 BC.

Baudhāyana Śulbasūtra, the oldest and biggest among the four, is divided into three chapters. In all, it contains 525 aphorisms. The first chapter gives various units of measures and some geometrical propositions. It also contains relative positions and spatial magnitude of various *vedi*s. The second chapter deals with construction of different *Agni*-s or *Agnicayana*, ie., the piling of bricks to form the sacrificial altar. The third chapter is devoted to the construction of seventeen kinds of *Kāmyāgni*-s. Two commentaries are available for this text. They are *Śulbadīpikā* of *Dvārakānāthayajvan* and *Śulbamīmāmsā* of *Veṅkaṭeśvara Dīkṣitar.*

The *Mānava Śulbasūtra* is composed in prose and verse. It is divided into seven *khaṇḍa*-s or chapters. The first chapter gives a description of the measuring tape, the gnomon, measures and four methods of determining the cardinal directions. Here it may be noted that *Bodhāyana* and *Āpastamba* do not give any such method of determining cardinal directions which is essential for the construction of sacrificial altars which are to be constructed symmetrically on the East-West line. The remaining chapters treat

relative positions, spatial magnitudes and method of construction of different *vedi*-s.

Āpastamba Śulbasūtra contains 223 aphorisms. It is divided into six *Paṭala*-s or sections. The first section gives the important geometrical propositions required for the construction of altars. The second section describes the relative positions of the various *Vedi*-s and their spatial magnitudes. And the third section deals with different kinds of *Kāmyāgni*-s.

There are four well known commentaries on *Āpastamba Śulbasūtra*. They are

1. *Śulbavyākhyā* of *Kapardisvāmin* (not later than 12th century)
2. *Śulbapradīpikā* of *Karavindasvāmin* (not later than 5th century)
3. *Śulbapradīpa* of *Sundararāja* (16th Century) and
4. *Āpastambīya Śulbabhāṣya* of *Gopala*, son of Gargya

Nṛsimhasomasutvan

Kātyāyanaśulbasūtra which is also known as *Katiya Śublbapariśiṣṭa* is divided into two parts. The first part written in *sūtra* style contains 90 *sūtra*s divided into seven *khaṇḍika*s or sections. The second part is composed in verses. The first part gives geometrical propositions and spatial magnitudes and relations for the construction of different *Agni*-s. This *Śulba* text does not deal with *Kāmyāgni*-s since it has been dealt with in the *Śrautasūtra* of the same author. The second part, in 48 verses, gives a description of measuring tape (*rajju*), the gnomon, the attributes of an expert altar-builder (*śulbavid*) and a few rules for his conduct. *Kātyāyana Śulbasūtra* carries much importance since in a succinct and systematic manner the text deals with the whole body of geometrical knowledge required for the vedic altar-builder. *Śulbasūtravṛtti* of

Rama/ Ramachandra (15th Century) and *Śulbasūtravivaraṇa* of *Mahīdhara* (16th Century) are the two known commentaries on *Kātyāyana Śulbasūtra*.

The origin and development of geometry

The *Yajussamhitā*-s and *Brāhmana*-s have laid down strict rules for different sacrifices, the violation of which is considered a great offence causing even negative effect. Strict injunctions are there in the *Vedas* which prescribe shape and area of different sacrificial alters. So the construction of such sacrificial altars according to the strict injunctions might have paved the way for origin and development of geometry the results of which are codified in the *śulba* texts. Many a time the shape is to be altered without altering the area. For instance, in the case of the three *nityāgni*-s described above, all the three should have the same area of one square *puruṣa*, whereas the shape differs. *Āhavanīya* should be circular; *Dakṣiṇā* semi-circular and the *Gārhapatya* of a square shape (some texts prescribe circular shape for *Gārhapatya* also). In the case of *Kāmyāgni*-s, as has already been noted, the standard form is *Śyenaciti* having an area of 7 ½ square *puruṣa*. All the other *Kāmyāgni*-s, in general are to have different shapes (some of which are cited above) with the same area. Thus transformation of different kinds of geometrical figures without altering area is the main concern of sacrificial altar constructors which lead to many geometrical propositions and findings. Here it may be noted that the transformation of a square into a circle and vice versa led the *śulbavid*-s to the evaluation of p, which they have done, though not with a high degree of accuracy.

As we have already seen the table of measures used, the geometrical propositions used and the construction of different shapes of sacrificial altars are the common features of all the *śulba* texts. A list of some important measures of length as prescribed by *Baudhayana* is given below.

1 *aṅgula*	=	34 *tila*-s (approximately 3/4 inches)
1 *pradeśa*	=	12 *aṅgula*-s
1 *pāda*	=	15 *aṅgula*-s
1 *bāhu*	=	36 *aṅgula*-s
1 *prakrama*	=	2 *pāda*-s
1 *aratni*	=	2 *pradeśa*-s = 24 *aṅgula*-s
1 *puruṣa*	=	5 *aratni*-s = 120 *aṅgula*-s
1 *vyama*	=	5 *aratni*-s

The important geometrical propositions the knowledge of which is implied in the *Śulbasūtra*-s.

1. A given set line can be divided into any number of equal parts.
2. A circle can be divided into any number of parts by drawing diameters.
3. Each diagonals of the rectangle bisect it.
4. The diagonals of a rectangle bisect each other and they divide the rectangle into four parts.
5. The diagonals of a rhombus bisect each other at right angles.
6. An isosceles triangle is divided into two equal halves by the line joining the vertex with the middle point of opposite side.
7. A triangle formed by joining the extremities of any side of a square to the middle point of opposite side to/////// equal to half the square.

8. A quadrilateral formed by the lines joining the middle points of the sides of a square is a square whose area is half that of the original one
9. A quadrilateral formed by the lines joining the middle points of the sides of a rectangle is a rhombus whose are a is half that of the rectangle.
10. A parallelogram and a square which are on the same base within the same parallels are equal to one another.
11. The maximum square that can be described within the circle is the one which has its corners on the circumference of the circle.

"2 and Pythagoras theorem.

The most remarkable achievements of the vedic *śulbavid*-s (sacrificial altar builders) as reflected in *śulba* texts are

1. the precise statement of the theorem of the square of the diagonal (Pythagoras theorem as is known today)
2. evaluation of "2 to a very high degree of accuracy and
3. the realization of its ("2) irrationality.

Theorem of square of the diagonal.

The 48th sutra of the 1st chapter of Bodhāyana sutra runs as follows: दीर्घचतुरश्रस्याक्ष्णयारज्जुः पार्श्वमानी तिर्यङ्मानी च यत्पृथग्भूते कुरुतस्तदुभयं करोति । It literally means the diagonal of a rectangle produces both (the areas) which its length and breadth produce separately. That means: The areas (of the squares) produced separately by the length and breadth of a rectangle together equal the area (of the square) produced by the diagonal. This is the same as the so-called Pythagoras theorem but put in a different phraseology. *Baudhāyana*

has repeated the statement with regard to square also since the concept of right angled triangle was not probably existent then. The statement runs as follows: समचतुरश्रस्याक्ष्णयारज्जुर्द्विस्तावतीं भूमिं करोति। (I-9). It means: The diagonal of a square produces double the area (of the square itself) .

Now this theorem is known after the name of Pythagoras (c.540 BC). But there is no clear evidence to show that he discovered the theorem. It was centuries after his death, tradition started attributing this theorem to him. It was Cicero (c.50 BC) who associated the name of Pythagoras with the theorem for the first time. *Bodhāyana* does not give the proof. Also it is not known as to how vedic Hindus arrived at the result. Historians have proposed different hypotheses regarding this.

Evaluation of "2

Bodhāyana, Āpastamba and *Kātyāyana* give the approximate value of "2 as 1+1/3 +1/(3x4) – 1/(3x4x34) which works out to 1.4142156 . It can be seen that this value bears a high degree of accuracy. *Bodhāyana*-s statement runs as follows:

प्रमाणं तृतीयेन वर्धयेत्तच्च चतुर्थेनात्मचतुस्त्रिंशोनेन। सविशेषः। (I -61-2) It means that the diagonal of a square having 'a' as its length, equals a + a/3 + a/(3x4) - a/(3x4x34).

ie. "2a = a + a/3 + a/(3x4) - a/(3x4x34) which gives the value of "2 as 1+1/3 +1/(3x4) – 1/(3x4x34). Bodhāyana adds the word *saviśeṣaḥ* which means 'with a residue'. That is, the value is not exact. It is implied that exact value cannot be found. Or in other words, it is implied that 2 is irrational. *Āpastamba* and *Kātyāyana* also give the same value with nearly identical words. *Kātyāyana* is more explicit in stating that the value will always have a residue. His

statement runs as : करणी तृतीयेन वर्धयेत्तच्च स्वचतुर्थेनात्मचतुस्त्रिंशोनेन सविशेष इति विशेष& (II/13). B. B. Dutta has shown that the commentators *Kapardiswamin* and *Karavindaswamin* have pointed out that the word *saviśeṣaḥ* meant 'always having an excess'. That means the exact value cannot be determined. Thus it can be seen that the vedic *śulbavid*-s knew the irrationality of "2.

Thus from the foregoing brief survey it can be seen that the vedic sacrificers had developed the science of geometry to a remarkable extent. And that the *Śulba* texts, the codified book forms of scattered knowledge of Vedic geometry can verily be called as the source book of Vedic Geometry.

Books referred to

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