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Intuitionistic Fuzzy Nano Topological Space: Theory and Applications

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Abstract

We introduce the notion of intuitionistic fuzzy nano topological space, its properties and results. The topological characterizations of intuitionistic fuzzy nano continuous functions are derived, and the weak forms of intuitionistic fuzzy nano-open sets are obtained. The intuitionistic fuzzy Nano Upper approximation space in real life application is discussed.

Keywords: Intuitionistic Fuzzy Nano Forms of Weakly Open Sets, Intuitionistic Fuzzy Nano Topological Space, Multi Criterion Decision Making

MSC (2010): Primary: 54B05, Secondary: 54C05, 08A72, 15B15, 68T37

1. Introduction

M. Lellis Thivagar and Carmel Richard[7-8] introduced nano topological space with respect to a subset *X* of an universe *U*. We have investigated the notion of Intuitionistic Fuzzy nano topological space[2-3-4-5, 9-10-11-12-13-14-15-16, 18].

2. Intuitionistic Fuzzy Nano Topological Space

Definition 3.1

Let *U* be a non-empty, finite universe of objects and *R* be an *IF* equivalence relation on *U*. Let A ⊆ U. Let $\tau_R(X) = \{1\sim, 0\sim, IFL_R(X), IFU_R(X), IFB_R(X)\}$. Then $\tau_R(X)$ satisfies the axioms of topology. i.e., $\tau_R(X)$ is a topology on *U* called the *IF* nano topology on *U* with respect to *A*. The elements of $\tau_R(X)$ are called as intuitionistic fuzzy nano-open sets (*IFNOS*, for short). In this case, the pair $(U, \tau_R(X))$ is called as intuitionistic fuzzy nano topological space (*IFNTS*, for short). In this regard, we refer[1,7-8,17].

Proposition 2.2

Let (U, R) be an *IF* approximation space (*IFAS*, for short), *C* and *D* subsets of *U* then

$$IFL_R(C) \subseteq C \subseteq IFU_R(C)$$

$$IFL_R(0\sim) = IFU_R(0\sim) = 0\sim$$

$$IFL_R(1\sim) = IFU_R(1\sim) = 1\sim$$

$$IFU_R(C \cup D) = IFU_R(C) \cup IFU_R(D)$$

$$IFU_R(C \cap D) \subseteq IFU_R(C) \cap IFU_R(D)$$

$$IFL_R(C \cup D) \supseteq IFL_R(C) \cup IFL_R(D)$$

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$$IFL_R(C \cap D) = IFL_R(C) \cap IFL_R(D)$$

- $IFL_R(C) \subseteq IFL_R(D)$ and $IFU_R(C) \subseteq IFU_R(D)$ whenever $C \subseteq D$
- $IFU_R(C^C) = [IFL_R(C)]^C$
- and $IFL_R(C^C) = [IFU_R(C)]^C$

$$IFU_R(IFU_R(C)) = IFL_R(C).$$

$$IFL_R(IFU_R(C)) = IFU_R(C).$$

Example 2.3

Let (U,R) be an *IFAS* where $U = \{d,e,f\}$ and $R \in R(U\times U)$ is defined as follows:

$R = \{\langle(d,d), 1\sim, 0\sim\rangle, \langle(d,e), 0.3, 0.4\rangle, \langle(e,d), 0.3, 0.4\rangle, \langle(e,e), 1\sim, 0\sim\rangle, \langle(e,f), 0.4, 0.5\rangle, \langle(f,e), 0.4, 0.5\rangle, \langle(f,f), 1\sim, 0\sim\rangle, \langle(d,f), 0.4, 0.3\rangle, \langle(f,d), 0.4, 0.3\rangle\}$

Let $A = \{d,0.7,0.3\rangle, \langle e,0.6,0.4\rangle, \langle f,0.6,0.4\rangle\}$ be an *IFS* on U then by definition, we have

$$IFU_R(A) = \{x, \mu_{IFU_R(A)}(x), \nu_{IFU_R(A)}(x) / x \in U\}$$

$$IFL_R(A) = \{x, \mu_{IFL_R(A)}(x), \nu_{IFL_R(A)}(x) / x \in U\}$$

Then,

$IFU_R(A) = \{\langle d,0.7,0.3\rangle, \langle e,0.6,0.4\rangle, \langle f,0.6,0.3\rangle\}$

$IFL_R(A) = \{\langle d,0.6,0.4\rangle, \langle e,0.6,0.4\rangle, \langle f,0.6,0.4\rangle\}$

$IFB_R(A) = \{\langle d,0.4,0.6\rangle, \langle e,0.4,0.6\rangle, \langle f,0.4,0.6\rangle\}$

$$\tau_R(X) = \{1\sim, 0\sim, IFL_R(X), IFU_R(X), IFB_R(X)\}$$

Remark 2.4[1,8,17]

Elements of $[\tau_R(A)]^C$ are called IF nano closed sets (*IFNCS*, for short).

Definition 2.5

If $(U, \tau_R(X))$ be an *IFNTS* in A, then the *IF* nano interior of A is defined as the union of all *IFNOS* contained in A and is denoted by *IFNInt*(*A*).

i.e., $IFNInt(A) = \cup\{G{:}G$ is an *IFNOS* in U and $G \subseteq A\}$.

i.e., *IFNInt*(*A*) is the largest nano open subset of A[1,18,17].

Definition 2.6

The *IF* nano closure of *A* is defined as the intersection of all *IF* closed subsets containing *A* and is denoted by *IFNCl*(*A*).

i.e., $IFNCl(A) = \cup\{K{:}K$ is an *IFNCS* in U and $A \subseteq K\}$.

i.e., *IFNCl*(*A*) is the smallest *IFNCS* containing A[1,18,17].

Definition 2.7

Let $(U,\tau_R(X))$ and $(V,\sigma_R(Y))$ be two IFNTSs. Then mapping f: $U \to V$ is an *IF* nano continuous (*IFNC*, for short) on U if the inverse image of every *IFNOS* in V is *IFNOS* in U.[1,8,17]

Example 2.8

Let (U,R) be an *IFAS* where $U = \{a,b,c\}$ with

$R = \{\langle(a,a),1\sim,0\sim\rangle, \langle(a,b),0.3, 0.3\rangle, \langle(b,a),0.3,0.3\rangle, \langle(b,b),1\sim,0\sim\rangle,$

$\langle(b,c),0.2,0.3\rangle, \langle(c,b),0.2,0.3\rangle, \langle(c,c),1\sim,0\sim\rangle, \langle(a,c),0.3,0.2\rangle, \langle(c,a),0.3,0.2\rangle\}$. $\{\langle(a,a),1\sim,0\sim\rangle,\langle(a,b),0.3,0.3\rangle,\langle(b,a),0.3,0.3\rangle, \langle(b,b),1\sim,0\sim\rangle, \langle(b,c),0.2,0.3\rangle,\langle(c,b),0.2,0.3\rangle,\langle(c,c),1\sim,0\sim\rangle,\langle(a,c),0.3,0.2\rangle,\langle(c,a),0.3,0.2\rangle\}$

Let $X = \{\langle a,0.2,0.1\rangle, \langle b,0.2,0.3\rangle, \langle c,0.3,0.2\rangle\}$ be an *IFS* on U then

$\tau_R(X) = \{1\sim, 0\sim, \{\langle a, 0.3, 0.1\rangle, \langle b, 0.2, 0.3\rangle, \langle c, 0.3, 0.2\rangle\}, \{\langle a, 0.2, 0.3\rangle, \langle b, 0.2, 0.3\rangle, \langle c, 0.2, 0.3\rangle\}, \{\langle a, 0.3, 0.2\rangle, \langle b, 0.2, 0.3\rangle, \langle c, 0.2, 0.3\rangle\}$.

Let (V,R) be an *IFAS* where $V = \{x,y,z\}$ with

$R = \{\langle(x,x),1\sim,0\sim\rangle, \langle(x,y),0.5,0.4\rangle, \langle(y,x),0.5,0.4\rangle, \langle(y,y),1\sim,0\sim\rangle,$

$\langle(y,z), 0.3, 0.4\rangle, \langle(z,y), 0.3, 0.4\rangle, \langle(z,z), 1\sim, 0\sim\rangle, \langle(x,z), 0.4, 0.5\rangle, \langle(z,x), 0.4, 0.5\rangle\}$.

Let $Y = \{\langle x,0.8,0.2\rangle, \langle y,0.7,0.3\rangle, \langle z,0.7,0.3\rangle\}$be an *IFS* on V then

$\sigma_R(Y) = \{1\sim, 0\sim, \langle x, 0.8, 0.2\rangle, \langle y, 0.7, 0.3\rangle, \langle z, 0.7, 0.3\rangle\}, \{\langle x, 0.7, 0.3\rangle, \langle y, 0.7, 0.3\rangle, \langle z, 0.7, 0.3\rangle\}, \{\langle x, 0.3, 0.7\rangle, \langle y, 0.3, 0.7\rangle, \langle z, 0.3, 0.7\rangle\}$.

Define $f: (U,\tau_R(X)) \to (V,\sigma_R(Y))$ as $f(a) = x$, $f(b) = y$, $f(c) = z$. Then the inverse image of every *IFNOS* in V is *IFNOS* in U.

Definition 2.9

A function f is said to be an *IF* nano homeomorphism (*IFNH*, for short) if f is

- f is one-one and onto.

- *f* is *IFNC*.
- *f* is *IFNOS*.

Theorem 2.10

Let ***f***: $(U,\tau_R(X)) \rightarrow (V,\sigma_R(Y))$ be a one-one onto mapping, then *f* is *IFNH* if and only if *f* is *IF* nano closed and *IFNC*.

Proof

Let *f* be a *IFNH*. Then *f* is *IFNC*. Let *S* be an arbitrary *IFNCS* in $(U,\tau_R(X))$. Then *U*–*S* is *IFNOS*. Since *f* is *IFNOS*, *f(U–S)* is *IFNOS* in *V*. That is, *V–f(S)* is *IFNOS* in *V*. Therefore, *f(S)* is *IF* nano closed in *V*. Thus, the image of every *IFNCS* in *U* is *IFNCS* in *V*. That is, *f* is *IFNCS*. Conversely, let *f* be *IF* nano closed and *IFNC*. Let B be *IFNOS* in $(U,\tau_R(X))$. Then *U–B* is *IFNCS* in *U*. Since *f* is *IF* nano closed, *f(U–B)* = *V–f(B)* is *IF* nano closed in *V*. Therefore, *f(B)* is *IFNOS* in V. Thus, *f* is *IFNOS* and hence *f* is a *IFNH*.

Theorem 2.11

A one-one function *f* of $(U,\tau R(X))$ onto $(V,\sigma R(Y))$ is a IFNH if and only if *f(IFNCl(A))* = *IFNCl(f(A))* for every subset *A* of *U*.

Proof

If *f* is an *IFNH*, *f* is *IFNC* and *IF* nano closed. If $A \subseteq U$, *f(IFNCl(A))* ⊆ *IFNCl(f(A))* since *f* is an *IFNC*. Since *IFNCl(A)* is an *IF* nano closed in *U* and *f* is *IF* nano closed, *f(IFNCl(A))* is *IF* nano closed in *V*. Therefore *IFNCl(f(IFNCl(A)))* = *f(IFNCl(A))*. Since $A \subseteq$ *IFNCl(A)*, *f(A)* ⊆ *f(IFNCl(A))* and hence *IFNCl(f(A))* ⊆ *IFNCl(f(IFNCl(A)))* = *f(IFNCl(A))*. Thus *IFNCl(f(A))* = *f(IFNCl(A))* if *f* is *IFNH*. Conversely, if *IFNCl(f(A))* = *f(IFNCl(A))* for every subset *A* of *U*, then *f* is IFNC. If *A* is *IF* nano closed in *U*, *A* = *IFNCl(A)* which implies *f(A)* = *f(IFNCl(A))*. Therefore, *IFNCl(f(A))* = *f(A)*. Thus, *f(A)* is an *IF* nano closed. Also *f* is *IFNC*. Therefore *f* is an *IFNH*.

3. *IF* Nano Forms of Weakly Open Sets

Let $(U,\tau_R(X))$ be an *IF* nano topological space (IFNTS) with respect to *X* where $X \subseteq U$, *R* is an equivalence relation on *U*.

$\frac{U}{R}$ denotes the family of equivalence classes of *U* by *R*.

Definition 3.1

Let $(U,\tau_R(X))$ be an *IFNTS* and $B \subseteq U$. Then *B* is said to be

- *IF* nano semi-open if
  $B \subseteq IFNCl(IFNInt(B))$
- *IF* nano pre-open if
  $B \subseteq IFNInt(IFNCl(B))$
- *IF* nano α–open if
  $B \subseteq IFNInt(IFNCl(IFNInt(B)))$
- *IF* regular open if
  $B = IFNInt(IFNCl(B))$

IFNSO, *IFNPO*, *IFN*α-open and *IFRO* respectively denote the families of all *IFNSO*, *IF* nano pre-open, *IF* nano α–open and *IF* regular open subsets of *U*.

Example 3.2

Let *(U,R)* be an *IFAS* where *U* = {*a,b,c*} and *R*∈ *R*(*U*×*U*) is defined as follows:

R = {⟨(*a,a*),1~,0~⟩, ⟨(*a,b*),0.3, 0.4⟩, ⟨(*b,a*),0.3,0.4⟩,⟨(*b, b*),1~,0~⟩, ⟨(*b,c*), 0.4, 0.5⟩, ⟨(*c,b*), 0.4, 0.5⟩, ⟨(*c,c*), 1~, 0~⟩, ⟨(*a,c*), 0.4, 0.3⟩, ⟨(*c,a*), 0.4, 0.3⟩}.

Let *X* = {⟨*a*,0.7,0.3⟩, ⟨*b*,0.6,0.4⟩, ⟨*c*,0.6,0.4⟩} be an *IF* set on *U* then by definition, we have

$\tau_R(A)$ = {1~,0~,{⟨*a*,0.7,0.3⟩, ⟨*b*,0.6,0.4⟩, ⟨*c*,0.6,0.3⟩}, {⟨a,0.6,0.4⟩, ⟨b,0.6,0.4⟩, ⟨c,0.6,0.4⟩}, {⟨a,0.4,0.6⟩, ⟨b,0.4,0.6⟩, ⟨c,0.4,0.6⟩}}.

Let *A* = {⟨*a*,0.7,0.3⟩, ⟨*b*,0.6,0.4⟩, ⟨*c*,0.6,0.3⟩}, then the *IFSA* is an *IFNSO* in *U*.

Let *A* = {⟨*a*,0.6,0.4⟩, ⟨*b*,0.6,0.4⟩, ⟨*c*,0.6,0.4⟩}, then the *IFSA* is an *IFNPO* in *U*.

Let *A* = {⟨*a*,0.7,0.3⟩, ⟨*b*,0.6,0.4⟩, ⟨*c*,0.6,0.3⟩}, then the *IFSA* is an *IFN*α-open in *U*.

Let *A* = {⟨*a*,0.6,0.4⟩, ⟨*b*,0.6,0.4⟩, ⟨*c*,0.6,0.4⟩}, then the *IFSA* is an *IFNRO* in *U*.

Theorem 3.3

If *A* is *IFNO* in $(U,\tau_R(X))$, then it is *IFN*α-open in *U*.

Proof

Since A is *IFNO* in U, *IFNInt(A)* = *A*.

Then *IFNCl (IFNInt(A))* = *IFNCl (A)* ⊇ *A*. That is, *A* ⊂ *IFNCl (IFNInt(A))*. Therefore, *IFNInt (A)* ⊆ *IFNInt (IFNCl(IFNInt(A)))*. That is, *A*⊆*IFNInt (IFNCl(IFNInt(A)))*. Thus, A is *IFN*α-open.

Theorem 3.4

$\tau_R^{\alpha}(X) \subseteq IFNSO(U, X)$ in a *IFNTS(U,τR(X))*.

Proof

If $A \in \tau_R^{\alpha}(X)$, $A \subseteq IFNInt(IFNCl(IFNInt(A))) \subseteq IFNCl(IFNInt(A))$ and hence $A \in IFNSO(U, X)$.

Theorem 3.5

$\tau_R^{\alpha}(X) \subseteq IFNSO(U, X)$ in a $IFNTS(U,\tau_R(X))$.

Proof

If $A \in \tau_R^{\alpha}(X)$, $A \subseteq IFNInt(IFNCl(IFNInt(A)))$. Since $IFNInt(A) \subseteq A$,, $IFNInt(IFNCl(IFNInt(A))) \subseteq IFNInt(IFNCl(A))$. That is, $A \subseteq IFNInt(IFNCl(A))$. That is, $\tau_R^{\alpha}(X) \subseteq IFNPO(U, X)$.

Theorem 3.6

If, in a $IFNTS(U,\tau R(X))$, $IFL_R(X) = IFU_R(X) = X$, then $1\sim$, $0\sim$,$IFL_R(X)(=IFU_R(X))$ and any set $A \supset IFL(X)$ are the only $IFN\alpha$-open sets in U.

Proof

Since $IFL_R(X) = IFU_R(X) = X$, the IF nano topology, $\tau_R(X) = \{1\sim,0\sim,IFL_R(X)\}$. Since any $IFNOS$ is IF nano α-open, $1\sim$, $0\sim$ and $IFL_R(X)$ are IF nano α–open in U. If $A \subset IFL_R(X)$, then $IFNInt(A) = 0\sim$, since $0\sim$ is the only $IFNO$ subset of A. Therefore $IFNCl(IFNInt(A)) = 0\sim$ and hence A is not IF nano α–open. If $A \supset IFL_R(X)$, $IFL_R(X)$ is the largest $IFNO$ subset of A and hence, $IFNInt(IFNCl(IFNInt(A))) = IFNInt(IFNCl(L_R(X))) = IFNInt(B_R(X)^C) = IFNInt(U)$, Since $IFB_R(X) = 0\sim$. Therefore, $IFNInt(IFNCl(IFNInt(A))) = U$ and hence, $A \subseteq IFNInt(IFNCl(IFNInt(A)))$. Therefore, A is $IFN\alpha$-open. Thus U, $0\sim$, $IFL_R(X)$ and any set $A \supset IFL_R(X)$ are the only $IFN\alpha$-open sets in U, if $IFL_R(X) = IFU_R(X)$.

Theorem 3.7

$1\sim$, $0\sim$, $IFU_R(X)$ and any set $A \supset IFU_R(X)$ are the only $IFN\alpha$-open sets in a $IFNTS(U,\tau_R(X))$, if $IFL_R(X) = 0\sim$.

Proof

Since $IFL_R(X) = 0\sim$, $IFB_R(X) = IFU_R(X)$.. Therefore,

$\tau_R(X) = \{1\sim,0\sim,IFU_R(X)\}$ and the members of $\tau_R(X)$ are $IFN\alpha$-open in U. Let $A \subset IFU_R(X)$. Then $IFNInt(A) = 0\sim$ and hence $IFNInt(IFNCl(IFNInt(A))) = 0\sim$. Therefore A is not $IFN\alpha$-open in U. If $A \supset IFU_R(X)$, then $IFU_R(X)$ is the largest $IFN\alpha$-open subset of A (unless, $IFU_R(X) = U$, in case of which $1\sim$ and $0\sim$ are the only nano-open sets in U). Therefore, $IFNInt(IFNCl(IFNInt(A))) = IFNInt(IFNCl(IFU_R(X))) = IFNInt(U)$ and hence $A \subseteq IFNInt(IFNCl(IFNInt(A)))$. Thus, any set $A \supset U_R(X)$ is $IFN\alpha$-open in U. Hence, $1\sim$, $0\sim$, $IFU_R(X)$ and any superset of $IFU_R(X)$ are the only $IFN\alpha$-open sets in U.

Corollary 3.8 $\tau_R(X) = \tau_R^{\alpha}(X)$, if $IFU_R(X) = U$.

Theorem 3.9

If, in a $IFNTS$ $(U,\tau_R(X))$, $IFU_R(X)=IFL_R(X)$, then $0\sim$ and set A such that $A \supseteq IFL_R(X)$ are the only $IFN\alpha$-open subsets of U.

Proof

$\tau_R(X) = \{1\sim,0\sim,IFL_R(X)\}$. $0\sim$ is $IFN\alpha$-open. If A is an non-empty subset of U and $A \subset IFL_R(X)$, then $IFNCl(IFNInt(A)) = IFNCl(0\sim) = 0\sim$. Therefore, A is not $IFN\alpha$-open, if $A \subset IFL_R(X)$. If $A \supseteq IFL_R(X)$, then $IFNCl(IFNInt(A)) = IFNCl(IFL_R(X)) = U$, since $IFU_R(X) = IFU_R(X)$. Therefore, $A \subseteq IFNCl(IFNInt(A))$ and hence A is $IFN\alpha$-open. Thus $0\sim$ and sets containing $IFL_R(X)$ are the only $IFN\alpha$-open sets in U, if $IFU_R(X) = IFL_R(X)$.

Theorem 3.10

Any $IFRO$ set is $IFNO$.

Proof

If A is $IFRO$ in $(U,\tau_R(X))$, $A = IFNInt(IFNCl(A))$. Then $IFNInt(A) = IFNInt(IFNInt(IFNCl(A))) = A$. That is, A is $IFNO$ in U.

Remark 3.11

The converse of the above theorem is not true. For example, let (U, R) be an IF approximation space where $U = \{a,b,c\}$ with

$R = \{\langle(a,a),1\sim,0\sim\rangle, \langle(a,b),0.3, 0.3\rangle, \langle(b,a),0.3,0.3\rangle, \langle(b,b),1\sim,0\sim\rangle, \langle(b,c),0.2,0.3\rangle, \langle(c,b),0.2,0.3\rangle, \langle(c,c),1\sim,0\sim\rangle, \langle(a,c),0.3,0.2\rangle, \langle(c,a),0.3,0.2\rangle\}$.

Let $X = \{\langle a,0.2,0.1\rangle, \langle b,0.2,0.3\rangle, \langle c,0.3,0.2\rangle\}$ be an IF set on U then

$\tau_R(X) = \{1\sim, 0\sim, \{\langle a,0.3,0.1\rangle, \langle b,0.2,0.3\rangle, \langle c,0.3,0.2\rangle\}, \{\langle a,0.2,0.3\rangle, \langle b,0.2,0.3\rangle, \langle c,0.2,0.3\rangle\}, \{\langle a,0.3,0.2\rangle, \langle b,0.2,0.3\rangle, \langle c,0.2,0.3\rangle\}\}$.

Let A = *{⟨a,0.3,0.1⟩, ⟨b,0.2,0.3⟩, ⟨c,0.3,0.2⟩}* and B = *{⟨a,0.2,0.2⟩, ⟨b,0.2,0.2⟩, ⟨c,0.2,0.2⟩}* be *IFRO* sets then $A \cup B$ = *{⟨a,0.3,0.1⟩, ⟨b,0.2,0.2⟩, ⟨c,0.3,0.2⟩}* is not *IFRO* set.

Theorem 3.12

In a $IFNTS(U,\tau_R(X))$, if $IFU_R(X) \neq IFL_R(X)$, then the only *IFRO* sets are $1\sim$, $0\sim$, $IFL_R(X)$ and $IFB_R(X)$.

Proof

The only *IFNOSs* in $(U,\tau_R(X))$ are $1\sim$, $0\sim$, $IFL_R(X)$, $IFU_R(X)$ and $IFB_R(X)$ and hence the only *IFCO* sets in U are $1\sim$, $0\sim$, $[IFL_R(X)]^C$, $[IFU_R(X)]^C$ and $[IFB_R(X)]^C$.

Case 1

Let $A = IFBR(X)$. Then $IFNCl(A) = [IFB_R(X)]^C$. Therefore, $IFNInt(IFNCl(A)) = IFNInt[IFB_R(X)]^C = [IFNCl(IFB_R(X))]^C = [(IFL_R(X))^C]^C = IFL_R(X) = A$. Therefore, $A = IFL_R(X)$ is *IFRO*.

Case 2

Let $A = IFB_R(X)$. Then $IFNCl(A) = [IFL_R(X)]^C$. Therefore, $IFNInt(IFNCl(A)) = IFNInt[IFL_R(X)]^C = [IFNCl(IFL_R(X))]^C$

$= [(IFB_R(X))^C]^C = IFB_R(X) = A$. Therefore, $A = IFB_R(X)$ is *IFRO*.

Case 3

Let $A = IFU_R(X)$. Then $IFNCl(A) = U$. Therefore, $IFNInt(IFNCl(A)) = IFNInt(U) = U \neq A$. That is, $IFUR(X) = A$ is not *IFRO* unless $IFU_R(X) = U$.

Case 4

Since $IFNInt(IFNCl(A)) = IFNInt(0\sim) = 0\sim$, $1\sim$ and $0\sim$ are *IFRO*. Also any *IFRO* is *IFNO*. Thus, $1\sim$, $0\sim$, $IFL_R(X)$ and $IFB_R(X)$ are the only *IFRO* sets.

4. A Real Life Application

We discuss a real life application of *IFNTS* on one or more universal sets to multi criterion decision making using *IFNUAS*. It is observed that in the case of insurance companies by investors due to various factors like affordable premium, quality of service, quaranteed returns, location of the company and various best products, investors depend on one or more insurance companies. Hence, *IF* relation provides the better relation between the investors and insurance companies.

Consider $V = \{v_1, v_2, v_3, v_4, v_5\}$, in which v_1 is affordable premium; v_2 is quality of service; v_3 is quaranteed returns; v_4 is location of the company; v_5 is various best products and decisions $U = \{u_1, u_2, u_3, u_4, u_5\}$, in which u_1 is excellent; u_2 is good; u_3 is satisfactory; u_4 is acceptable; u_5 is least acceptable. Investors from various financial status are invited to the survey. Therefore, (U, V, IFU_R, IFL_R) be an *IFAS*, where $U = \{u_1, u_2, u_3, u_4, u_5\}$ and $V = \{v_1, v_2, v_3, v_4, v_5\}$.

If 16% investors give excellent and 11% give not excellent; 26% give good; 21% give not good; 36% give satisfactory; 6% give not satisfactory; 11% give acceptable; 22% give not acceptable; 16% give least acceptable and 10% give not acceptable, then we have $(.16,.11; .26,.21; .36,.06; .11,.22; .16,.1)^t$. Similarly, for other criteria's: $(.56, .2; .16, .46; .3, .16; 0, .6; .3, .7)^t$, $(.21, .3; .36, .22; .26, .16; .2, .7; .2, .3)$ t, $(.1, .8; .2, .5; .5, .3; .3, .4; .3, .1)^t$ and $(0, .7; 0, .6; .16, .4; .3, .6, .2; .6, .15)^t$. Based on the decision vectors, the IF relation from U to V is given by the following matrix.

$$R_{IF} = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{array} \begin{array}{c} \begin{array}{ccccc} V_1 & V_2 & V_3 & V_4 & V_5 \end{array} \\ \begin{bmatrix} \langle 0.16, 0.11\rangle & \langle 0.56, 0.02\rangle & \langle 0.21, 0.03\rangle & \langle 0.01, 0.08\rangle & \langle 0.00, 0.07\rangle \\ \langle 0.26, 0.21\rangle & \langle 0.16, 0.46\rangle & \langle 0.36, 0.22\rangle & \langle 0.02, 0.05\rangle & \langle 0.00, 0.06\rangle \\ \langle 0.36, 0.06\rangle & \langle 0.03, 0.16\rangle & \langle 0.26, 0.16\rangle & \langle 0.05, 0.03\rangle & \langle 0.16, 0.04\rangle \\ \langle 0.11, 0.22\rangle & \langle 0.00, 0.06\rangle & \langle 0.02, 0.07\rangle & \langle 0.03, 0.04\rangle & \langle 0.36, 0.02\rangle \\ \langle 0.16, 0.01\rangle & \langle 0.03, 0.07\rangle & \langle 0.03, 0.07\rangle & \langle 0.03, 0.01\rangle & \langle 0.06, 0.15\rangle \end{bmatrix} \end{array}$$

Two category of investors are considered, where right weightage for each criterion in U are $U_1 = (<u_1,.36,.15>, <u_2,.16,.3>, <u_3,.3,.3>, <u_4,.2,0.5>, <u_5,.1,.4>)$ and $U_2 = (<u_1,0.21,0.32>, <u_2,0.16,0.42>, <u_3,0.2,0.4>, <u_4,0.2,0.4>, <u_5,0.2,0.3>)$ respectively. Thus, by using *IF* upper approximation we have:

$IFU_R(V_1) = (<u_1,0.21,0.15>, <u_2,0.3,0.21>, <u_3,0.36,0.15>, <u_4,0.3,0.22>, <u_5,0.2,0.15>)^t$ and $IFU_R(V_2) = (<u_1,0.21,0.3>, <u_2,0.36,0.22>, <u_3,0.26,0.2>, <u_4,0.2,0.3>, <u_5,0.2,0.3>)^t$ respectively.

From above, according to the principle of maximum membership, the decision for the first category of investors is satisfactory whereas for the second category it is good.

6. Conclusion

The main research is focused on introducing intuitionistic fuzzy nano topological space with some properties and its characterizations. We have investigated a real time problem in Multi Criterion Decision Making.

7. References

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