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Solving Fuzzy Differential Equations using R-K (Runge-Kutta) Method

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Abstract

In a wide variety of technical specialties as well as in the field of medical, *(fuzzy differential equations) FDEs* models being used. *Chang S.L. and Zadeh L.A.* developed the term "fuzzy derivative." After that, *Dubosis, D.*, and *Prade* applied the extension principle in their strategy. *Kandel A.* and *Byatt W.J.* coined the phrase "fuzzy differential equation" in 1987. This study applies the *Hukuhara* derivative of the fuzzy process to the R-K Method of order 5 to solve fuzzy first- and second-order ODEs. This approach has been demonstrated by resolving a fuzzy Cauchy problem, followed by the numerical elucidation of FDEs using the order of five of R-K method.

Keywords: Approximate Solution, Exact Equation, FDEs, Sequence, Simultaneous Equation

1. Introduction

FDEs can be seen as a sort of unclear differential equation wherein the uncertain readings of the parameters, coefficients, and boundaries are all taken into consideration as fuzzy numbers. The value given to a variable by a fuzzy number might be thought of as being imprecise. A fuzzy problem with differential equations can be thought of as a differential equation with unreliable values, or "granular values," as its partners, coefficients, and conditions. This viewpoint treats a fuzzy differential equations parameter, coefficients, and conditions as the degranulation values. Fuzzy number-valued functions $\tilde{q}:(a,b)\subseteq R\rightarrow E_1$ and $\tilde{q}:(a,b)\subseteq R\rightarrow E_2$ are connected to first type and second type fuzzy functions, respectively. Different fuzzy derivative definitions have formed the most significant portion of FDE history. Fuzzy derivatives can be divided into fractional order and integer order categories. Using the fifth order R-K Method, we are able to solve fuzzy differential equations of integer order.

Using a numerical method for n-th order fuzzy differential equations based on Seikkala derivative with initial value conditions is studied[1]. From a scheme based on the Taylor method of order p is discussed in detail[2]. The paper discussed the generalized concepts of differentiability (of any order $n\in N$)[3]. Set theoretical relations are obtained for fuzzy mappings, fuzzy functions, and fuzzy parametric functions[4]. Differentiation of ordinary functions at a fuzzy point with fuzzy differential calculus[5] are studied. Using a special kind of fuzzy linear programming problem involving symmetric trapezoidal fuzzy numbers are considered[6]. The solution of the fuzzy linear vertical infiltration equation is examined in [7]. [8] deals with fuzzy-set-valued mappings of a real variable whose values are normal, convex, upper semicontinuous.

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2. 2^{nd} Order FDE

Derivation 2^{nd} of order FDEs using fifth order R-K Method

Theorem: 3.1
Assume the 2^{nd} order FDEs

$$D'(w) = q\left(i, w, \frac{dw}{dt}\right)$$
$$w(t_0) = w_0, w'(t_0) = w_0'$$

Then the approximate solution of 2^{nd} order FDE using fifth order R-K Method is determined.

Proof:
Two first order concurrent FDEs can be derived from the given 2^{nd} order FDE as shown below.

$$D(v) = v = q_1(i, w, v), i \in [i_0, I]$$
$$D(v) = q_2(i, w, v)$$
$$w(t_0) = w_0,\ w'(t_0) = v(t_0) = v_0$$

Where w_0 and v_0 are fuzzy numbers.

These are the equations' precise answers:

$$(\breve{w}(i))_j = (\breve{w}_1(i;j), \breve{w}_2(i;j))$$
$$(\breve{v}(i))_r = (\breve{v}_1(i;j), \breve{v}_2(i;j))$$

The nearly correct answers to the above-mentioned equations are,

$$(\breve{w}(i))_j = (\breve{w}_1(i;j), \breve{w}_2(i;j))$$
$$(\breve{v}(i))_j = (\breve{v}_1(i;j), \breve{v}_2(i;j))$$

By the use of 5^{th} order R-K Method for ***t=0, 1,.....N*** the nearly solution is the following calculations:

$$\breve{w}_1[i_{t+1};r] = \breve{w}_1[i_t;r] + \mathrm{h}\sum_{r=1}^{5}\omega_r \Bbbk_{r,1}\left[i_{t,}\breve{w}(i_t;j), \breve{v}_1(i_t;j)\right]$$
$$\breve{w}_2[i_{t+1};r] = \breve{w}_2[i_t;r] + \mathrm{h}\sum_{r=1}^{5}\omega_r \Bbbk_{r,2}\left[i_{t,}\breve{w}(i_t;j), \breve{v}_1(i_t;j)\right]$$
$$\breve{v}_1[i_{t+1};r] = \breve{v}_1[i_t;r] + \mathrm{h}\sum_{r=1}^{5}\omega_r \mathbb{1}_{r,1}\left[i_{t,}\breve{w}(i_t;j), \breve{v}_1(i_t;j)\right]$$
$$\breve{v}_2[i_{t+1};r] = \breve{v}_2[i_t;r] + \mathrm{h}\sum_{r=1}^{5}\omega_r \mathbb{1}_{r,2}\left[i_{t,}\breve{w}(i_t;j), \breve{v}_1(i_t;j)\right]$$

where the ω_r are constants. Then $\Bbbk_{r,1}, \Bbbk_{r,2} \mathbb{1}_{r,1}$ and $\mathbb{1}_{r,2}$ for **r = 1, 2, 3, 4** consists of the following:

$$\Bbbk_{1,1}\left[i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right] = \min\left\{q(i_t, \mathrm{u}, \mathrm{v}) \mid \mathrm{u} \in \left(\left[\breve{w}_1(i_t;j), \breve{w}_2(i_t;j)\right], \mathrm{v} \in \left[\breve{v}_1(i_t;j), \breve{v}_2(i_t;j)\right]\right)\right\}$$

$$\Bbbk_{1,2}\left[i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right] = \max\left\{q(i_t, \mathrm{u}, \mathrm{v}) \mid \mathrm{u} \in \left(\left[\breve{w}_1(i_t;j), \breve{w}_2(i_t;j)\right], \mathrm{v} \in \left[\breve{v}_1(i_t;j), \breve{v}_2(i_t;j)\right]\right)\right\}$$

$$\Bbbk_{2,1}\left[i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right] = \min\left\{\begin{array}{l} q\left(i_t + \frac{\mathrm{h}}{3}, \mathrm{u}, \mathrm{v}\right) \mid u \in \left(p_{1,1}\left(i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right), p_{1,2}\left(i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right)\right), \\ v \in \left(q_{1,1}\left(i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right), q_{1,2}\left(i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right)\right) \end{array}\right\}$$

$$\Bbbk_{2,2}\left[i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right] = \max\left\{\begin{array}{l} q\left(i_t + \frac{\mathrm{h}}{3}, \mathrm{u}, \mathrm{v}\right) \mid u \in \left(p_{1,1}\left(i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right), p_{1,2}\left(i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right)\right), \\ v \in \left(q_{1,1}\left(i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right), q_{1,2}\left(i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right)\right) \end{array}\right\}$$

$$\Bbbk_{3,1}\left[i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right] = \min\left\{\begin{array}{l} q\left(i_t + \frac{\mathrm{h}}{3}, \mathrm{u}, \mathrm{v}\right) \mid u \in \left(p_{2,1}\left(i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right), p_{2,2}\left(i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right)\right), \\ v \in \left(q_{2,1}\left(i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right), q_{2,2}\left(i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right)\right) \end{array}\right\}$$

$$\Bbbk_{3,2}\left[i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right] = \max\left\{\begin{array}{l} q\left(i_t + \frac{\mathrm{h}}{3}, \mathrm{u}, \mathrm{v}\right) \mid u \in \left(p_{2,1}\left(i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right), p_{2,2}\left(i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right)\right), \\ v \in \left(q_{2,1}\left(i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right), q_{2,2}\left(i_{t,}\breve{w}(i_t;j), \breve{v}(i_t;j)\right)\right) \end{array}\right\}$$

$$\mathbb{k}_{4,1}\left[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right]=\min\left\{\begin{array}{l} q\left(i_t+\frac{\mathrm{h}}{2},\mathrm{u},\mathrm{v}\right)\mid u\in\left(p_{3,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),p_{3,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right),\\ v\in\left(q_{3,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),q_{3,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right)\end{array}\right\}$$

$$\mathbb{k}_{4,2}\left[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right]=\max\left\{\begin{array}{l} q\left(i_t+\frac{\mathrm{h}}{2},\mathrm{u},\mathrm{v}\right)\mid u\in\left(p_{3,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),p_{3,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right),\\ v\in\left(q_{3,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),q_{3,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right)\end{array}\right\}$$

$$\mathbb{k}_{5,1}\left[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right]=\min\left\{\begin{array}{l} q\left(i_t+\mathrm{h},\mathrm{u},\mathrm{v}\right)\mid u\in\left(p_{4,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),p_{4,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right),\\ v\in\left(q_{4,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),q_{4,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right)\end{array}\right\}$$

$$\mathbb{k}_{5,2}\left[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right]=\max\left\{\begin{array}{l} q\left(i_t+\mathrm{h},\mathrm{u},\mathrm{v}\right)\mid u\in\left(p_{4,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),p_{4,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right),\\ v\in\left(q_{4,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),q_{4,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right)\end{array}\right\}$$

$$\mathbb{l}_{1,1}\left[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right]=\min\left\{q(i_t,\mathrm{u},\mathrm{v})\mid \mathrm{u}\in\left(\left[\breve{w}_1(i_t;j),\breve{w}_2(i_t;j)\right],\mathrm{v}\in\left[\breve{v}_1(i_t;j),\breve{v}_2(i_t;j)\right]\right)\right\}$$

$$\mathbb{l}_{1,2}\left[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right]=\max\left\{q(i_t,\mathrm{u},\mathrm{v})\mid \mathrm{u}\in\left(\left[\breve{w}_1(i_t;j),\breve{w}_2(i_t;j)\right],\mathrm{v}\in\left[\breve{v}_1(i_t;j),\breve{v}_2(i_t;j)\right]\right)\right\}$$

$$\mathbb{l}_{2,1}\left[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right]=\min\left\{\begin{array}{l} q\left(i_t+\frac{\mathrm{h}}{3},\mathrm{u},\mathrm{v}\right)\mid u\in\left(p_{1,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),p_{1,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right),\\ v\in\left(q_{1,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),q_{1,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right)\end{array}\right\}$$

$$\mathbb{l}_{2,2}\left[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right]=\max\left\{\begin{array}{l} q\left(i_t+\frac{\mathrm{h}}{3},\mathrm{u},\mathrm{v}\right)\mid u\in\left(p_{1,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),p_{1,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right),\\ v\in\left(q_{1,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),q_{1,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right)\end{array}\right\}$$

$$\mathbb{l}_{3,1}\left[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right]=\min\left\{\begin{array}{l} q\left(i_t+\frac{\mathrm{h}}{3},\mathrm{u},\mathrm{v}\right)\mid u\in\left(p_{2,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),p_{2,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right),\\ v\in\left(q_{2,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),q_{2,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right)\end{array}\right\}$$

$$\mathbb{l}_{3,2}\left[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right]=\max\left\{\begin{array}{l} q\left(i_t+\frac{\mathrm{h}}{3},\mathrm{u},\mathrm{v}\right)\mid u\in\left(p_{2,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),p_{2,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right),\\ v\in\left(q_{2,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),q_{2,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right)\end{array}\right\}$$

$$\mathbb{l}_{4,1}\left[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right]=\min\left\{\begin{array}{l} q\left(i_t+\frac{\mathrm{h}}{2},\mathrm{u},\mathrm{v}\right)\mid u\in\left(p_{3,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),p_{3,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right),\\ v\in\left(q_{3,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),q_{3,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right)\end{array}\right\}$$

$$\mathbb{l}_{4,2}\left[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right]=\max\left\{\begin{array}{l} q\left(i_t+\frac{\mathrm{h}}{2},\mathrm{u},\mathrm{v}\right)\mid u\in\left(p_{3,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),p_{3,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right),\\ v\in\left(q_{3,1}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right),q_{3,2}\left(i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)\right)\right)\end{array}\right\}$$

$$\mathrm{l}_{5,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\min\left\{\begin{array}{l} q\left(i_t+\mathrm{h},\mathrm{u},\mathrm{v}\right)|\ u\in\left(p_{4,1}\left(i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right),p_{4,2}\left(i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right)\right),\\ v\in\left(q_{4,1}\left(i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right),q_{4,2}\left(i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right)\right)\end{array}\right\}$$

$$\mathrm{l}_{5,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\max\left\{\begin{array}{l} q\left(i_t+\mathrm{h},\mathrm{u},\mathrm{v}\right)|\ u\in\left(p_{4,1}\left(i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right),p_{4,2}\left(i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right)\right),\\ v\in\left(q_{4,1}\left(i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right),q_{4,2}\left(i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right)\right)\end{array}\right\}$$

where in the R-K Method of order five, the variable $\mathrm{p}_{t,r}$ and $\mathrm{q}_{t,r}$ are defined as follows

$$\mathrm{p}_{1,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\breve{w}_1\left(i_t;j\right)+\frac{1}{3}\mathbb{k}_{1,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]$$

$$\mathrm{p}_{1,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\breve{w}_2\left(i_t;j\right)+\frac{1}{3}\mathbb{k}_{1,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]$$

$$\mathrm{p}_{2,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\breve{w}_1\left(i_t;j\right)+\frac{1}{6}\mathbb{k}_{1,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]+\frac{1}{6}\mathbb{k}_{2,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]$$

$$\mathrm{p}_{2,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\breve{w}_2\left(i_t;j\right)+\frac{1}{6}\mathbb{k}_{1,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]+\frac{1}{6}\mathbb{k}_{2,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]$$

$$\mathrm{p}_{3,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\breve{w}_1\left(i_t;j\right)+\frac{1}{8}\mathbb{k}_{1,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]+\frac{3}{8}\mathbb{k}_{3,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]$$

$$\mathrm{p}_{3,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\breve{w}_2\left(i_t;j\right)+\frac{1}{8}\mathbb{k}_{1,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]+\frac{3}{8}\mathbb{k}_{3,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]$$

$$\mathrm{p}_{4,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\breve{w}_1\left(i_t;j\right)+\frac{1}{2}\mathbb{k}_{1,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]-\frac{3}{2}\mathbb{k}_{3,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]+2\mathbb{k}_{4,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]$$

$$\mathrm{p}_{4,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\breve{w}_2\left(i_t;j\right)+\frac{1}{2}\mathbb{k}_{1,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]-\frac{3}{2}\mathbb{k}_{3,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]+2\mathbb{k}_{4,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]$$

$$\mathrm{q}_{1,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\breve{v}_1\left(i_t;j\right)+\frac{1}{3}\mathrm{l}_{1,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]$$

$$\mathrm{q}_{1,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\breve{v}_2\left(i_t;j\right)+\frac{1}{3}\mathrm{l}_{1,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]$$

$$\mathrm{q}_{2,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\breve{v}_1\left(i_t;j\right)+\frac{1}{6}\mathrm{l}_{1,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]+\frac{1}{6}\mathrm{l}_{2,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]$$

$$\mathrm{q}_{2,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\breve{v}_1\left(i_t;j\right)+\frac{1}{6}\mathrm{l}_{1,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]+\frac{1}{6}\mathrm{l}_{2,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]$$

$$\mathrm{q}_{3,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\breve{v}_1\left(i_t;j\right)+\frac{1}{8}\mathrm{l}_{1,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]+\frac{3}{8}\mathrm{l}_{3,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]$$

$$\mathrm{q}_{3,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\breve{v}_2\left(i_t;j\right)+\frac{1}{8}\mathrm{l}_{1,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]+\frac{3}{8}\mathrm{l}_{3,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]$$

$$\mathrm{q}_{4,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\breve{v}_1\left(i_t;j\right)+\frac{1}{2}\mathrm{l}_{1,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]-\frac{3}{2}\mathrm{l}_{3,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]+2\mathrm{l}_{4,1}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]$$

$$\mathrm{q}_{4,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]=\breve{v}_2\left(i_t;j\right)+\frac{1}{2}\mathrm{l}_{1,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]-\frac{3}{2}\mathrm{l}_{3,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]+2\mathrm{l}_{4,2}\left[i_{t,}\breve{w}\left(i_t;j\right),\breve{v}\left(i_t;j\right)\right]$$

Now, using the initial conditions w_0 and v_0 and the 5th -order R-Kformula, we calculate

$$\breve{w}_1[i_{t+1};r]=\breve{w}_1(i_t;j)+\frac{\mathrm{h}}{6}\Big[\mathbb{k}_{1,1}[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)]+4\mathbb{k}_{4,1}[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)]+\mathbb{k}_{5,1}[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)]\Big]$$

$$\breve{w}_2[i_{t+1};r]=\breve{w}_2(i_t;j)+\frac{\mathrm{h}}{6}\Big[\mathbb{k}_{1,2}[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)]+4\mathbb{k}_{4,2}[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)]+\mathbb{k}_{5,2}[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)]\Big]$$

$$\breve{v}_1[i_{t+1};r]=\breve{v}_1[i_t;r]+\frac{\mathrm{h}}{6}\Big[\mathrm{l}_{1,1}[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)]+4\mathrm{l}_{4,1}[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)]+\mathrm{l}_{5,1}[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)]\Big]$$

$$\breve{v}_1[i_{t+1};r]=\breve{v}_1[i_t;r]+\frac{\mathrm{h}}{6}\Big[\mathrm{q}_{1,2}[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)]+4\mathrm{l}_{4,2}[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)]+\mathrm{l}_{5,2}[i_{t,}\breve{w}(i_t;j),\breve{v}(i_t;j)]\Big]$$

The convergence of the approximate solution to the precise solution is demonstrated by the following equations

$$\lim_{\mathrm{h}\to 0}\breve{w}_1(i;j)=\breve{w}_1(i;j)$$
$$\lim_{\mathrm{h}\to 0}\breve{w}_2(i;j)=\breve{w}_2(i;j)$$
$$\lim_{\mathrm{h}\to 0}\breve{v}_1(i;j)=\breve{v}_1(i;j)$$
$$\lim_{\mathrm{h}\to 0}\breve{v}_2(i;j)=\breve{v}_2(i;j)$$

Theorem : 3.2
Let the number list go $\{\mathrm{m}_\mathrm{n}\}_{\mathrm{n}=0}^{\mathbb{N}}$ satisfy

$$|\mathrm{m}_\mathrm{n}+1|\le\mathbb{C}|\mathrm{m}_\mathrm{n}|+\mathbb{D},0\le\mathrm{n}\le\mathbb{N}-1$$

Given a positive constants $\mathbb{C}$ and $\mathbb{D}$. Then

$$|\mathrm{m}_\mathrm{n}|\le\mathbb{C}^\mathrm{n}|\mathrm{m}_0|+\mathbb{D}\frac{\mathbb{C}^\mathrm{n}-1}{\mathbb{C}-1},0\le\mathrm{n}\le\mathbb{N}$$

Prove that the domain containing the elements of $\mathbb{F}_1,\mathbb{G}_1,\mathbb{F}_2$ and $\mathbb{G}_2$ satisfies the equation

$$|\mathrm{m}_\mathrm{n}|\le\mathbb{C}^\mathrm{n}|\mathrm{m}_0|+\mathbb{D}\frac{\mathbb{C}^\mathrm{n}-1}{\mathbb{C}-1},0\le\mathrm{m}\le\mathbb{N}$$

Proof:
Generally, replacing the value of i_t with i and after replacing $\breve{w}_1(i;j)$,$\breve{v}_1(i;j)$ by $\breve{\mathrm{u}},\breve{\mathrm{v}}$ then $\mathbb{F}_1(i,\breve{\mathrm{u}},\breve{\mathrm{m}})$,

$\mathbb{G}_1(i,\breve{\mathrm{u}},\breve{\mathrm{m}})$,$\mathbb{F}_2(i,\breve{\mathrm{u}},\breve{\mathrm{m}})$,$\mathbb{G}_2(i,\breve{\mathrm{u}},\breve{\mathrm{m}})$ are defined as follows:

$$\mathbb{F}_1(i,\breve{\mathrm{u}},\breve{\mathrm{m}})=\mathbb{k}_{1,1}(i,\breve{\mathrm{u}},\breve{\mathrm{m}})+4\mathbb{k}_{4,1}(i,\breve{\mathrm{u}},\breve{\mathrm{m}})+\mathbb{k}_{5,1}(i,\breve{\mathrm{u}},\breve{\mathrm{m}})$$
$$\mathbb{G}_1(i,\breve{\mathrm{u}},\breve{\mathrm{m}})=\mathbb{k}_{1,2}(i,\breve{\mathrm{u}},\breve{\mathrm{m}})+4\mathbb{k}_{4,2}(i,\breve{\mathrm{u}},\breve{\mathrm{m}})+\mathbb{k}_{5,2}(i,\breve{\mathrm{u}},\breve{\mathrm{m}})$$

$$\mathbb{F}_2(i,\breve{u},\breve{m}) = l_{1,1}(i,\breve{u},\breve{m}) + 4l_{4,1}(i,\breve{u},\breve{m}) + l_{5,1}(i,\breve{u},\breve{m})$$
$$\mathbb{G}_2(i,\breve{u},\breve{m}) = l_{1,2}(i,\breve{u},\breve{m}) + 4l_{4,2}(i,\breve{u},\breve{m}) + l_{5,2}(i,\breve{u},\breve{m})$$

The domain containing the elements of $\mathbb{F}_1, \mathbb{G}_1, \mathbb{F}_2, \mathbb{G}_2$ are defined by

$$K = \left([i,\breve{u},\breve{m}] \mid 0 \le t \le \mathbb{T}, -\infty < \dot{u} < \infty, -\infty < \dot{m} < \infty \right)$$

Therefore, the domain $\mathbb{K}$ satisfies the given equation by containing the elements o f $\mathbb{F}_1, \mathbb{G}_1, \mathbb{F}_2$ and $\mathbb{G}_2$.

Theorem :3.3
Let the list of numbers go $\{m_n\}_{n=0}^{\mathbb{N}}, \{s_n\}_{n=0}^{\mathbb{N}}$ satisfy

$$|m_{n+1}| \le |m_n| + \mathbb{C}.\max\{|m_n|, |s_n|\} + \mathbb{D}$$
$$|s_{n+1}| \le |m_n| + \mathbb{C}.\max\{|m_n|, |s_n|\} + \mathbb{D}$$

Given a positive constant $\mathbb{C}$ and $\mathbb{D}$ **,** and denote

$$|u_n| = |m_n| + |s_n|, 0 \le n \le \mathbb{N}$$

Then $|u_n| \le \overline{\mathbb{C}^n}|u_0| + \overline{\mathbb{D}}\frac{\overline{\mathbb{C}^n} - 1}{c - 1}, 0 \le n \le \mathbb{N}$, where $\overline{\mathbb{C}} = +1 + 2\mathbb{C}$ and $\overline{\mathbb{D}} = 2\mathbb{D}$.

Prove that the domain containing the elements of $\mathbb{F}_1, \mathbb{G}_1, \mathbb{F}_2$ and $\mathbb{G}_2$ satisfies the equation

$$|\mathbb{U}_n| \le \overline{\mathbb{C}^n}|u_0| + \overline{\mathbb{D}}\frac{\overline{\mathbb{C}^n} - 1}{\mathbb{C} - 1}, 0 \le n \le \mathbb{N}$$

Proof:
Generally, replacing the value of i_t with i and after replacing $\breve{w}_1(i;j)$, $\breve{w}_2(i;j)$, $\breve{v}_1(i;j)$, and $\breve{v}_2(i;j)$

By i, $\breve{u}, \breve{s}, \breve{m}, \breve{v}$ then $F_1(i,\breve{u},\breve{s},\breve{m},\breve{v})$, $F_2(i,\breve{u},\breve{s},\breve{m},\breve{v})$, $G_1(i,\breve{u},\breve{s},\breve{m},\breve{v})$, $G_2(i,\breve{u},\breve{s},\breve{m},\breve{v})$, are defined as follows:

$$\mathbb{F}_1(i,\breve{u},\breve{s},\breve{m},\breve{v}) = \mathbb{k}_{1,1}(i,\breve{u},\breve{s},\breve{m},\breve{v}) + 4\mathbb{k}_{4,1}(i,\breve{u},\breve{s},\breve{m},\breve{v}) + \mathbb{k}_{5,1}(i,\breve{u},\breve{s},\breve{m},\breve{v})$$
$$\mathbb{G}_1(i,\breve{u},\breve{s},\breve{m},\breve{v}) = \mathbb{k}_{1,2}(i,\breve{u},\breve{s},\breve{m},\breve{v}) + 4\mathbb{k}_{4,2}(i,\breve{u},\breve{s},\breve{m},\breve{v}) + \mathbb{k}_{5,2}(i,\breve{u},\breve{s},\breve{m},\breve{v})$$
$$\mathbb{F}_2(i,\breve{u},\breve{s},\breve{m},\breve{v}) = l_{1,1}(i,\breve{u},\breve{s},\breve{m},\breve{v}) + 4l_{4,1}(i,\breve{u},\breve{s},\breve{m},\breve{v}) + l_{5,1}(i,\breve{u},\breve{s},\breve{m},\breve{v})$$
$$\mathbb{G}_2(i,\breve{u},\breve{s},\breve{m},\breve{v}) = l_{1,2}(i,\breve{u},\breve{s},\breve{m},\breve{v}) + 4l_{4,2}(i,\breve{u},\breve{s},\breve{m},\breve{v}) + l_{5,2}(i,\breve{u},\breve{s},\breve{m},\breve{v})$$

The domain containing the elements of $\mathbb{F}_1, \mathbb{G}_1, \mathbb{F}_2$ and $\mathbb{G}_2$ are defined by

$$K = \begin{pmatrix} [i,\breve{u},\breve{s},\breve{m},\breve{v}] \mid 0 \le i \le T, -\infty <, \breve{u} < \infty, -\infty < \breve{s} < \infty, \\ -\infty < \breve{m} < \infty, -\infty < \breve{v} < \infty \end{pmatrix}$$

Therefore, the domain $\mathbb{K}$ satisfies the given equation by containing the elements of $\mathbb{F}_1, \mathbb{G}_1, \mathbb{F}_2$ and $\mathbb{G}_2$

3. Conclusion

The first order and the second-order ordinary FDEs can be solved numerically using a fifth order R-K Method. Here, the second order FDE is divided into two simultaneous first order FDEs before the equations' solutions are computed. As a result, the precise values for this procedure are good and acceptable.

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